Principles of Astrometry

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astrometry, *n.*

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View as: Outline | Full entry   Quotations: Show all | Hide all Keywords: On | Off

Pronunciation: Brit. /əˈstrɒmɨtri/, U.S. /əˈstrɑmətri/

Frequency (in current use): ●●●○○○○○

Origin: Formed within English, by compounding. **Etymons:** ASTRO- *comb. form*, -METRY *comb. form*.

Etymology: < ASTRO- *comb. form* + -METRY *comb. form*. Compare French *astrométrie* (1845).

Astron.

The measurement and analysis of the position, motion, or magnitude of celestial objects, esp. stars.

Categories »

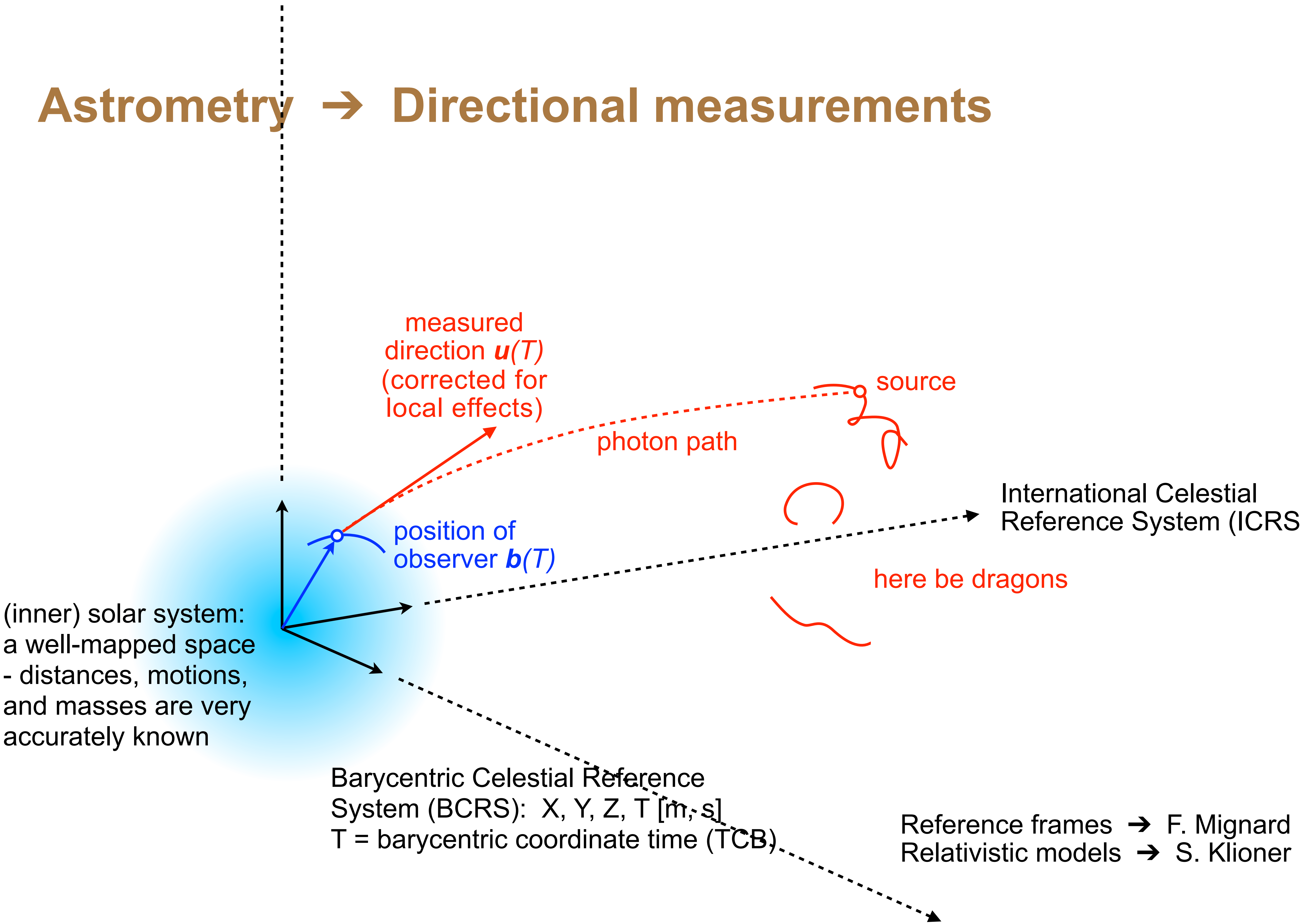
Quot. 1847 represents the introduction of the word in its modern technical use.

1811 *Philos. Mag.* **37** 45 No one has ever said that astronomy is astrometry or uranometry, although the heavens and the stars are measured.

1847 J. F. W. HERSCHEL *Results Astron. Observ.* iii. 304 Of astrometry, or the numerical expression of the apparent magnitudes of the stars.

OED Online (OED Third Edition, 2012), Oxford University Press

Astrometry → Directional measurements



Reference frames → F. Mignard
Relativistic models → S. Klioner

Source model (stellar and extragalactic objects)

“Source” = any sufficiently point-like object

Model: Constant space velocity in the barycentric system:

$$\boldsymbol{b}(T) = \boldsymbol{b}_0 + (T - T_{\rm ep})\boldsymbol{v}$$

$T_{\rm ep}$ = reference epoch (e.g. J2015.0 for TGAS)

Model has 6 kinematic parameters:

5 astrometric parameters

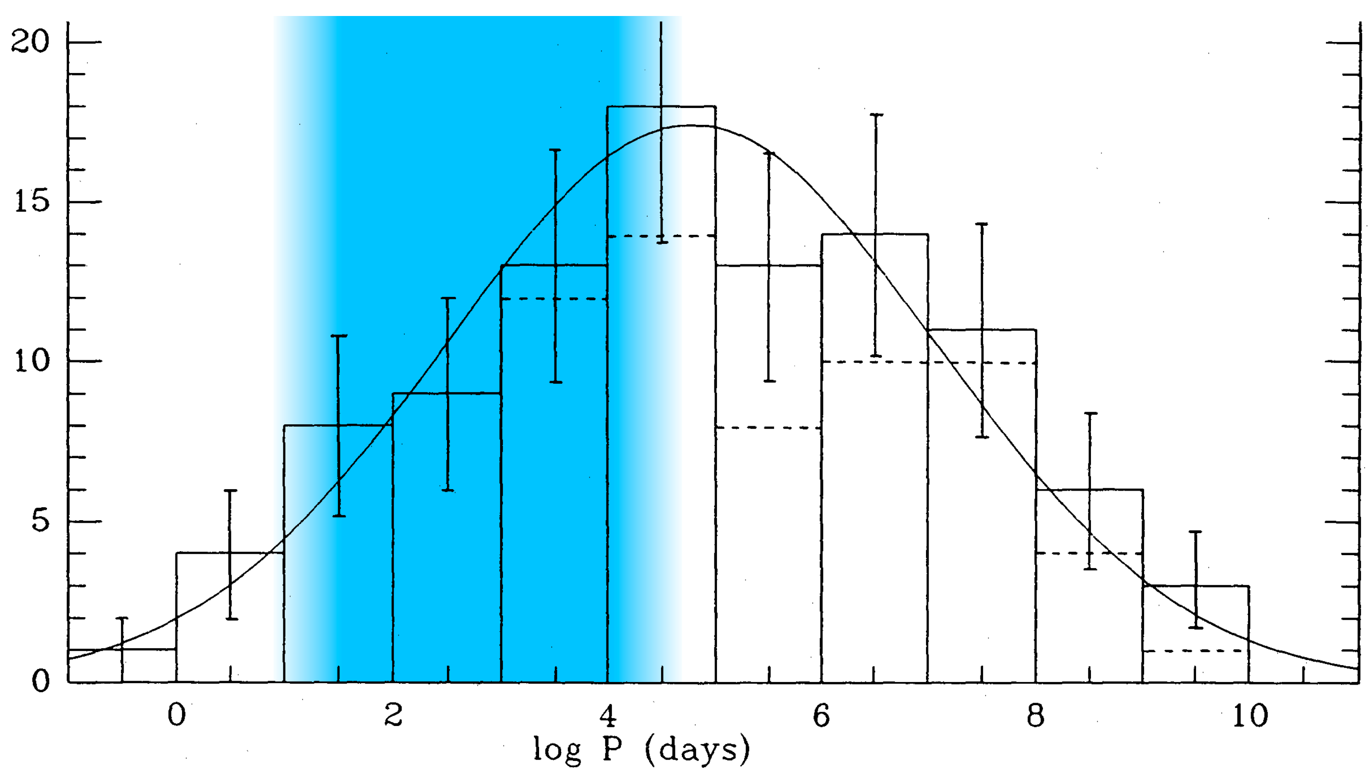
$$(b_{0X},\ b_{0Y},\ b_{0Z},\ v_X,\ v_Y,\ v_Z) \quad \Leftrightarrow \quad (\alpha,\ \delta,\ \varpi,\ \mu_{\alpha*},\ \mu_\delta,\ v_R)$$

For the modelling, v_R can be ignored except for some very nearby stars

➔ 5 astrometric parameters:
standard model for “single” stars, quasars, etc

Why is the 5-parameter model good enough?

- Galactic orbits are curved ➔ negligible
- Variable surface structures ➔ significant only for some (super)giants
- Most stars are members of double/multiple systems ➔ curved motion



Period distribution of G dwarf primaries (Duquennoy & Mayor, 1991):

50% have a stellar companion
log-normal P with median = 180 yr
and sigma = 2.3 dex

P < 10 d: orbit << parallax
P > 100 yr: curvature << parallax

40% of binaries have 10 d < P < 100 yr
➔ 20% of sources will be problematic

Instrument (calibration) models

or

The limits of self-calibration

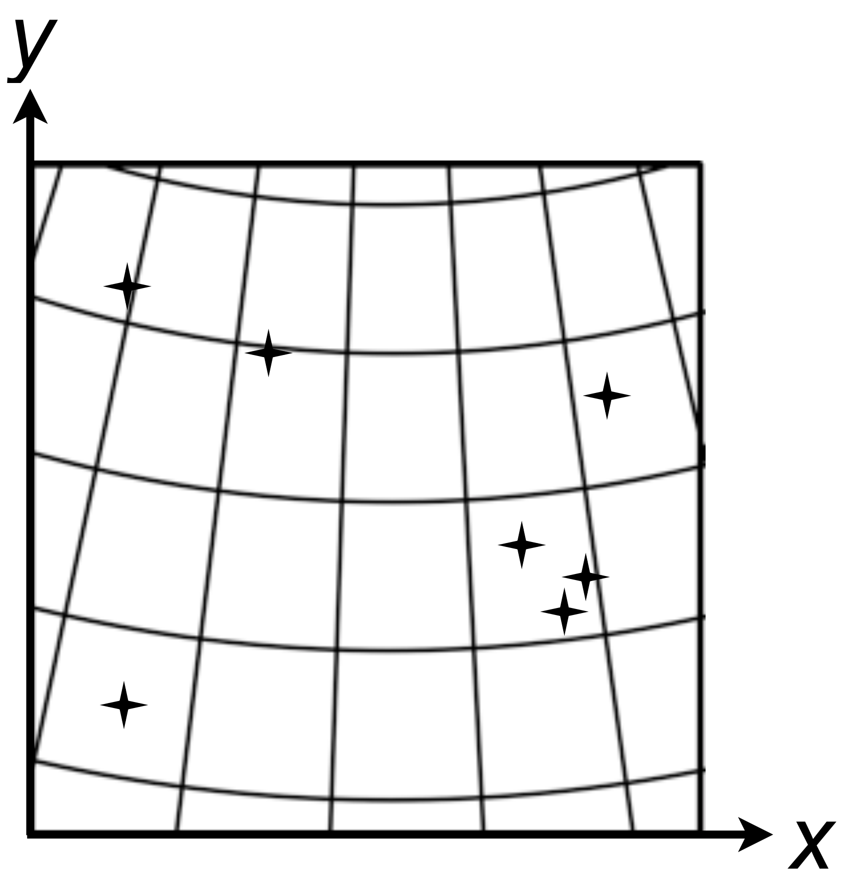
Calibration models

No universal model − depends entirely on the application:

- type of instrument
- wavelength region
- imaging or interferometric
- relative or absolute
- small-field or global
- space or ground-based
- ...

Example: Classical plate model

Used e.g. in photographic wide-field astrometry (AC, AGK2, AGK3, ...)

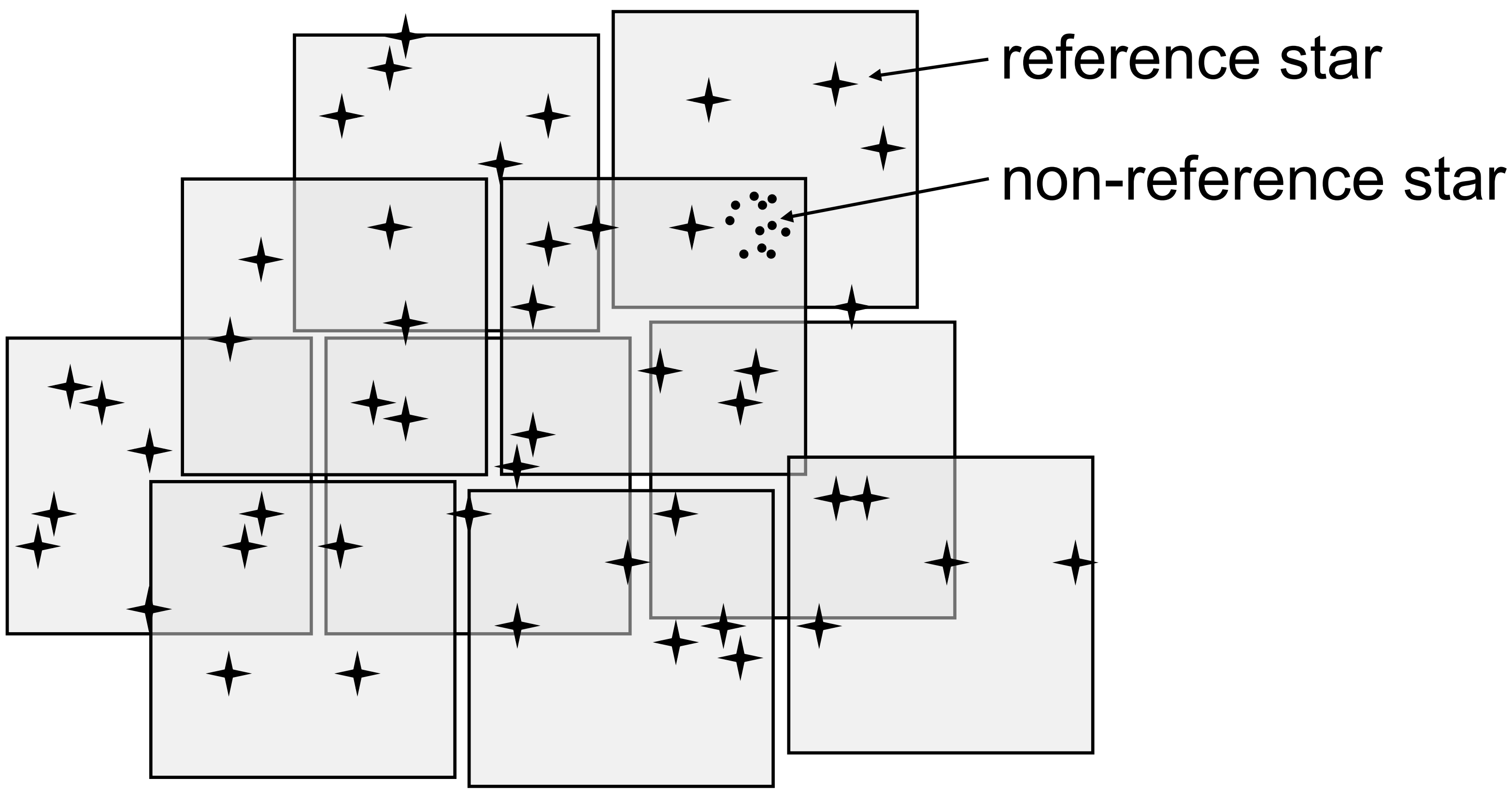


1. Measure plate coordinates (x, y) of all objects
2. Identify "reference stars" with known (α, δ)
3. Fit plate model $(x, y) \overset{f}{\longleftrightarrow} (\alpha, \delta)$ to the ref. stars
4. Apply f to measured (x, y) of the other objects

Problems:

- Low density of reference stars
- Higher-order models not possible
- Calibration not better than the reference stars

Plate-overlap technique (block adjustment)



Eichhorn (1960)

- Fit several overlapping plates simultaneously
- Every star measured on two or more plates gives additional constraints (for consistent α, δ)
- Need to solve large systems of equations

Self-calibration principle

1. Rely as little as possible on external "standards" − they are often not as good as your data!

2. Take multiple exposures of the same field at different times, orientation, etc.

3. Use parametrized models of sources (***s***) and other relevant factors, e.g. telescope pointing and distortion ("nuisance parameters", ***n***)

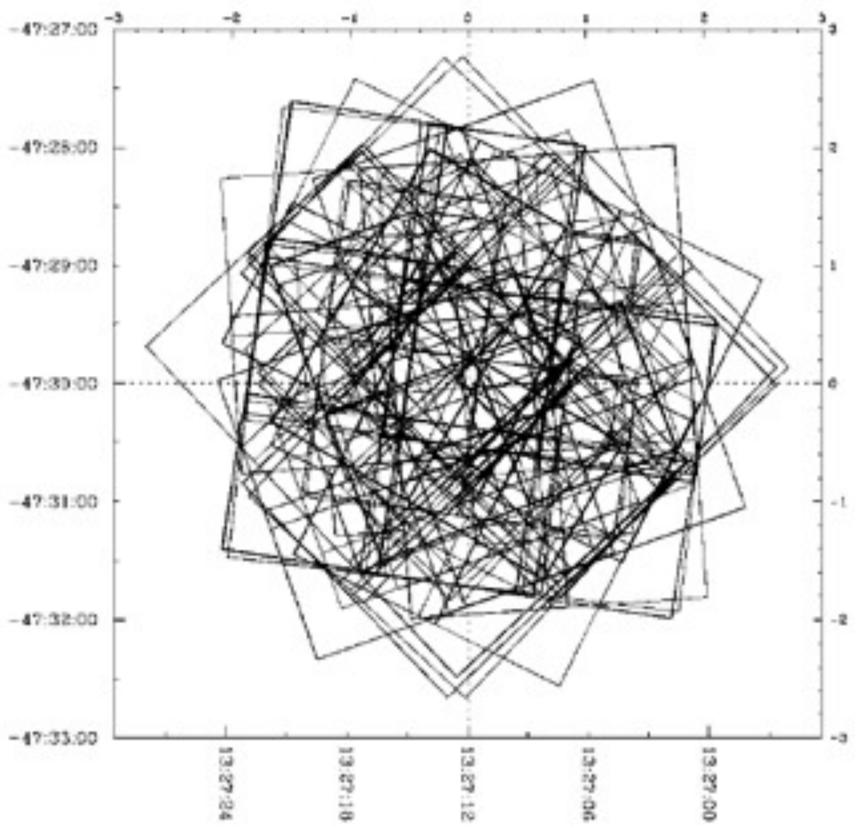
4. Solve the parameter values that best match the model (f) to the data:

$$\min_{\boldsymbol{s},\boldsymbol{n}} \left\|\mathrm{obs} - f(\boldsymbol{s},\boldsymbol{n})\right\|_{\mathcal{M}} \quad \Rightarrow \quad \boldsymbol{s},\, \boldsymbol{n}$$

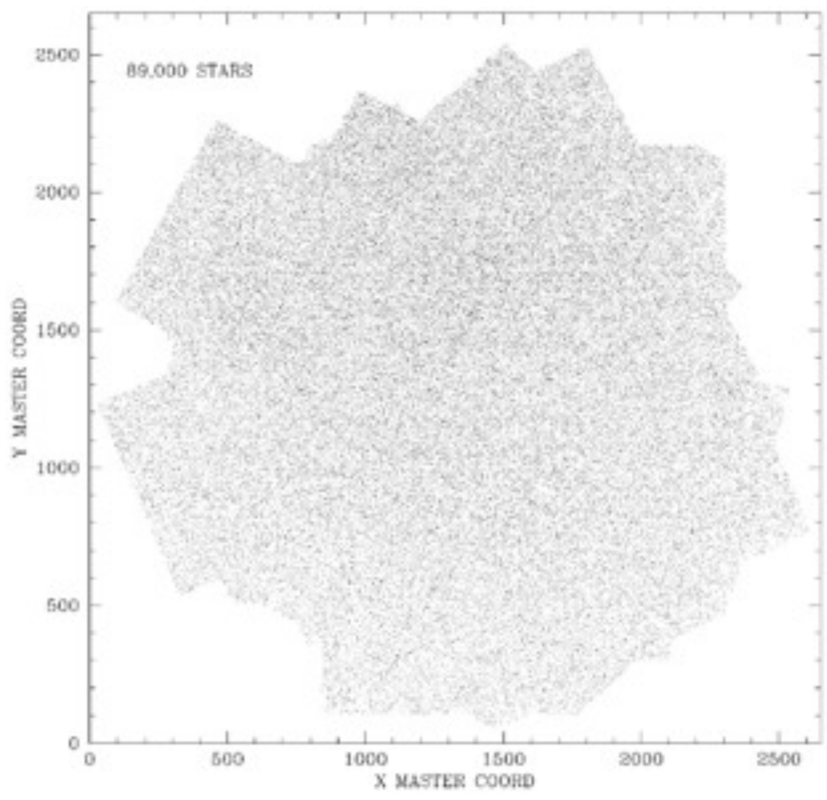
5. Usually, the solution is not unique ($\boldsymbol{s} \in S_f$ = solution space), and external standards may be used to select the preferred solution in S_f

Self-calibration example: HST cameras

- Anderson & King (2003) PASP 115, 113 (calibrating WFPC2 using ω Cen)



Pattern of exposures

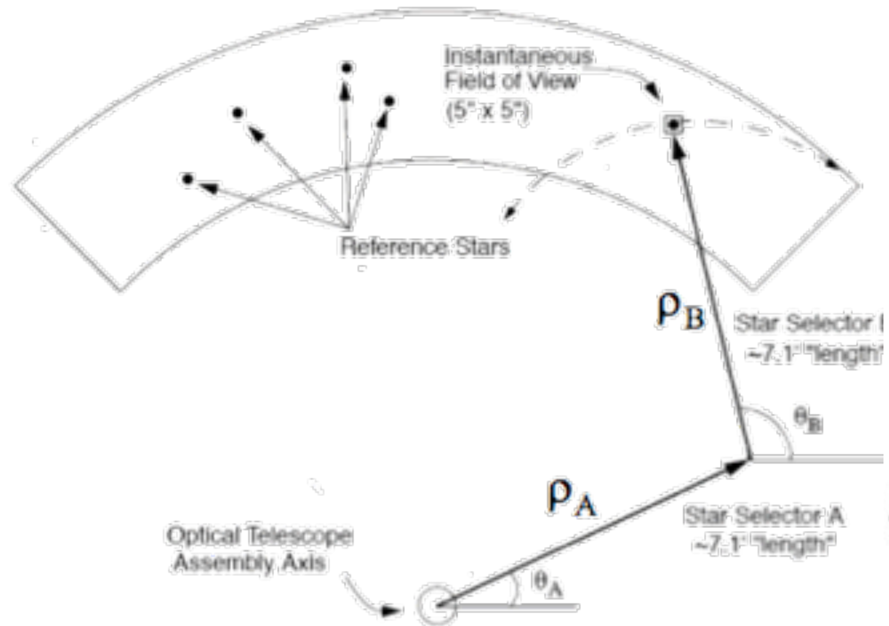


Map of 89,000 stars used

Self-calibration example: HST Fine Guidance Sensors (FGS)

Calibration parameters:

- ρ_A, ρ_B (arm lengths)
- κ_A, κ_B (offsets in θ_A, θ_B)
- a_{ij}, b_{ij} (distortion)



$$
\begin{aligned}
x' = {} & a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{11}xy + a_{30}x(x^2+y^2) + a_{21}x(x^2-y^2) \\
& + a_{12}y(y^2-x^2) + a_{03}y(y^2+x^2) + a_{50}x(x^2+y^2)^2 + a_{41}y(y^2+x^2)^2 \\
& + a_{32}x(x^4-y^4) + a_{23}y(y^4-x^4) + a_{14}x(x^2-y^2)^2 + a_{05}y(y^2-x^2)^2
\end{aligned}
$$

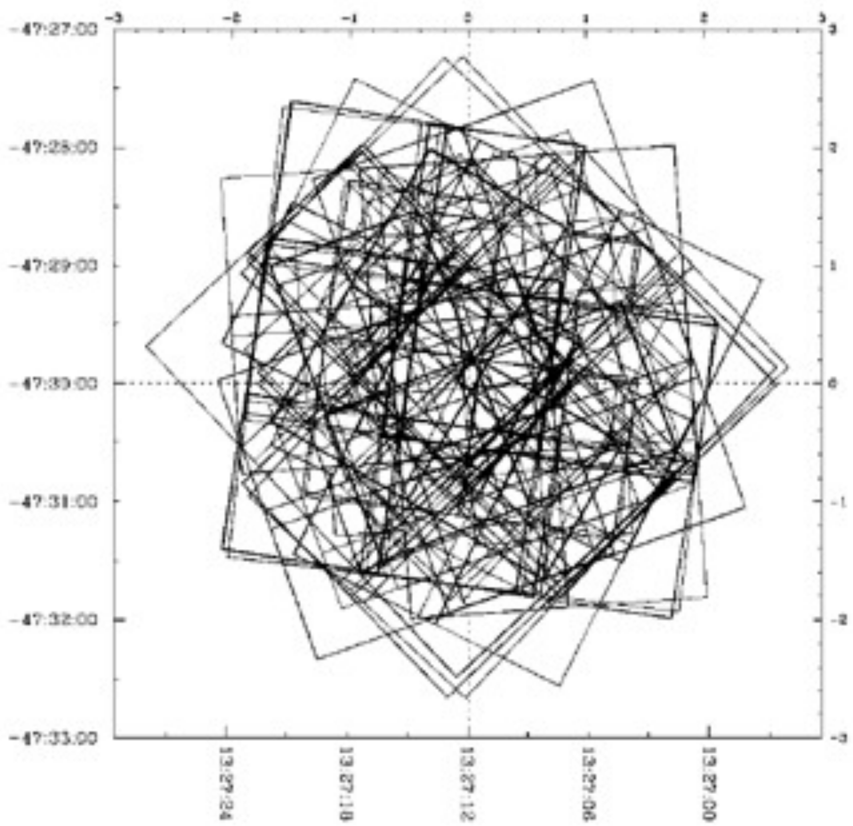
$$
\begin{aligned}
y' = {} & b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{02}y^2 + b_{11}xy + b_{30}x(x^2+y^2) + b_{21}x(x^2-y^2) \\
& + b_{12}y((y^2-x^2) + b_{03}y(y^2+x^2) + b_{50}x(x^2+y^2)^2 + b_{41}y(y^2+x^2)^2 \\
& + b_{32}x((x^4-y^4) + b_{23}y(y^4-x^4) + b_{14}x(x^2-y^2)^2 + b_{05}y(y^2-x^2)^2
\end{aligned}
$$

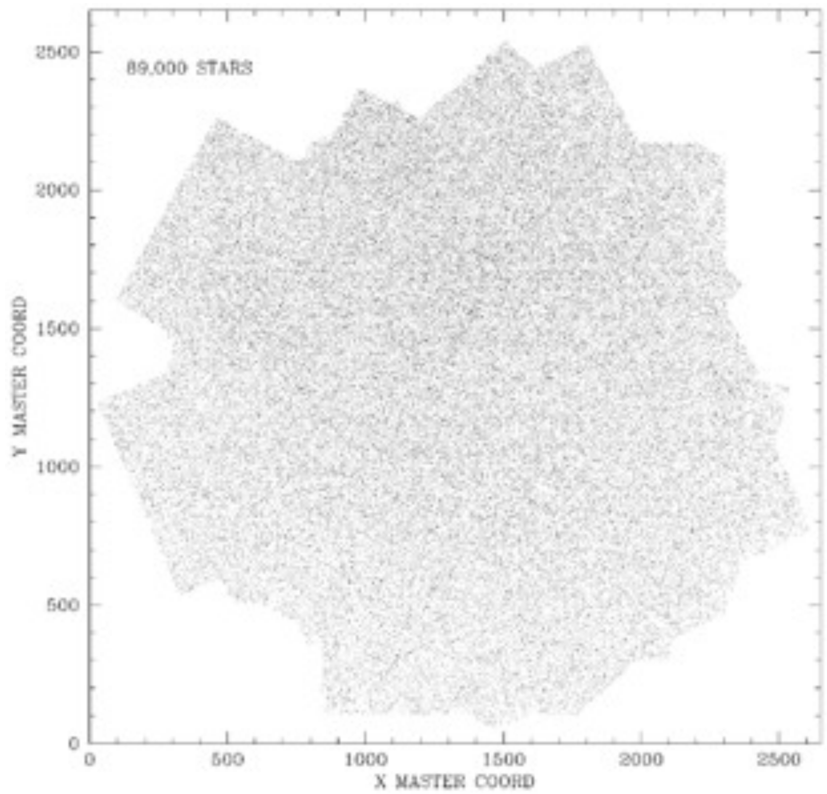
Figure 1. Rotation and Offsets of FGS 1R Winter 2000 OFAD.

Calibration field in M35
(McArthur, Benedict & Jefferys, 2002)

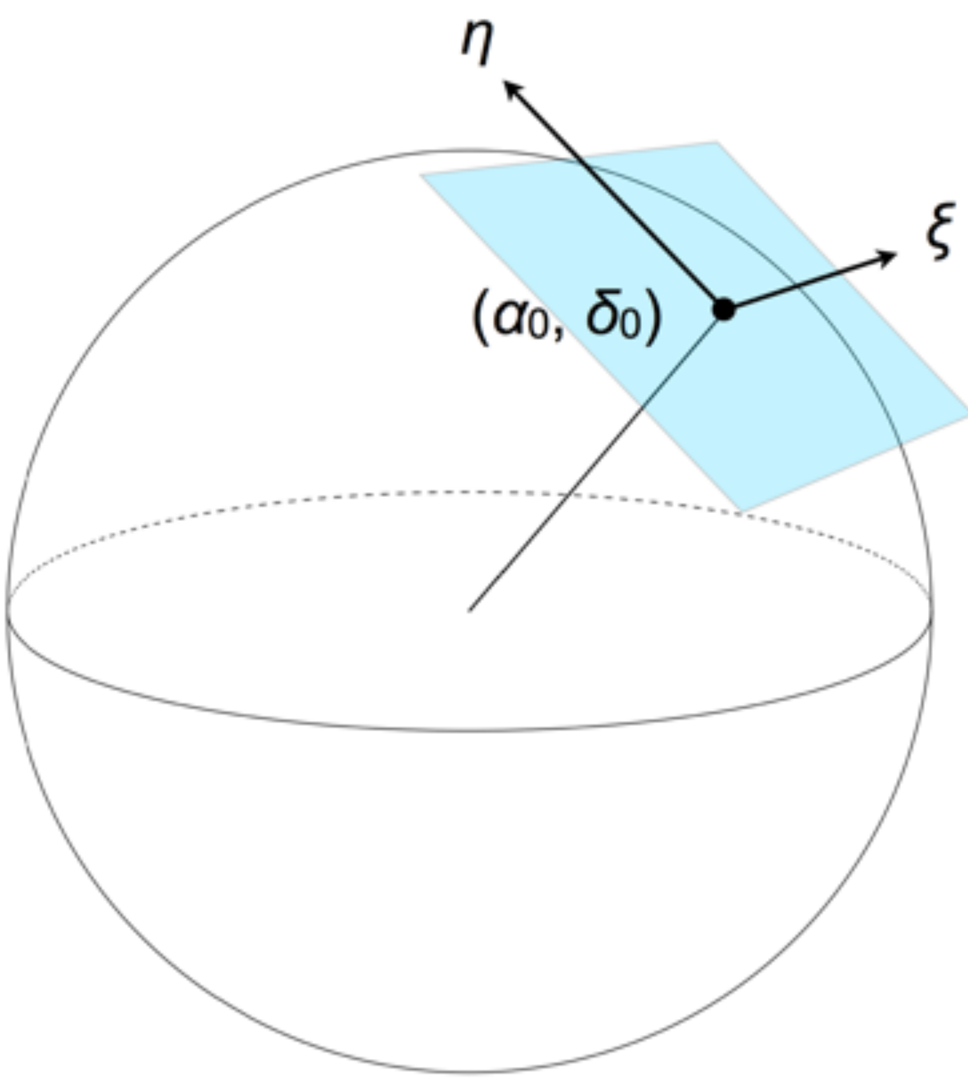
A simple toy model for illustration

- Superficially resembling the HST camera calibration





Toy model: Source



Neglecting v_R the 5-parameter model is linear in tangential coordinates ξ, η (gnomonic projection):

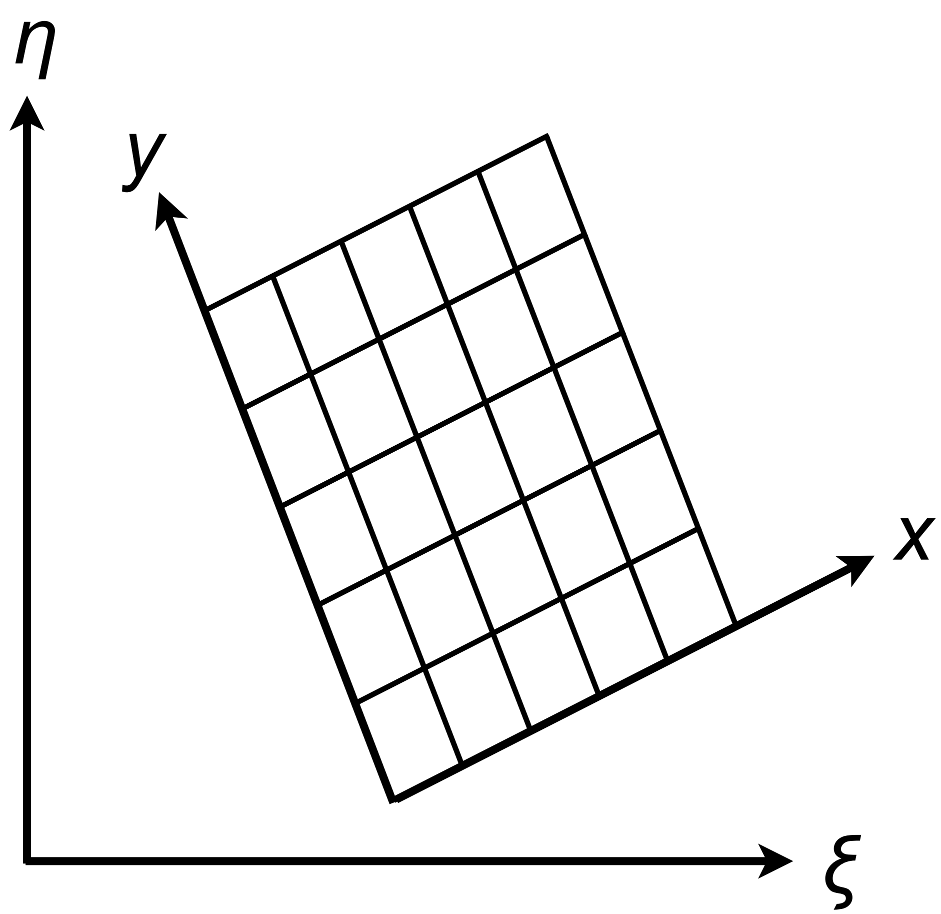
$$\xi(t) = a + bt + \varpi\Pi_\xi$$

$$\eta(t) = d + et + \varpi\Pi_\eta$$

Π_ξ, Π_η = known parallax factors (assumed constant over the field)

➔ 5 parameters per source: *a*, *b*, *d*, *e*, ϖ

Toy model: Calibration



Assume the most general linear relation between

- tangent plane coordinates (ξ, η) and
- pixel coordinates (x, y) :

$$x = A + B\xi + C\eta$$

$$y = D + E\xi + F\eta$$

➔ 6 parameters per exposure:
A, *B*, *C*, *D*, *E*, *F*

Toy model: Synthesis

M stars (i = 1...M) in N exposures (j = 1...N) ➔ 2MN non-linear equations:

$$x_{ij} = A_j + B_j(a_i + b_i t_j + \omega_i \Pi_{\xi j}) + C_j(d_i + e_i t_j + \omega_i \Pi_{\eta j})$$
$$y_{ij} = D_j + E_j(a_i + b_i t_j + \omega_i \Pi_{\xi j}) + F_j(d_i + e_i t_j + \omega_i \Pi_{\eta j})$$

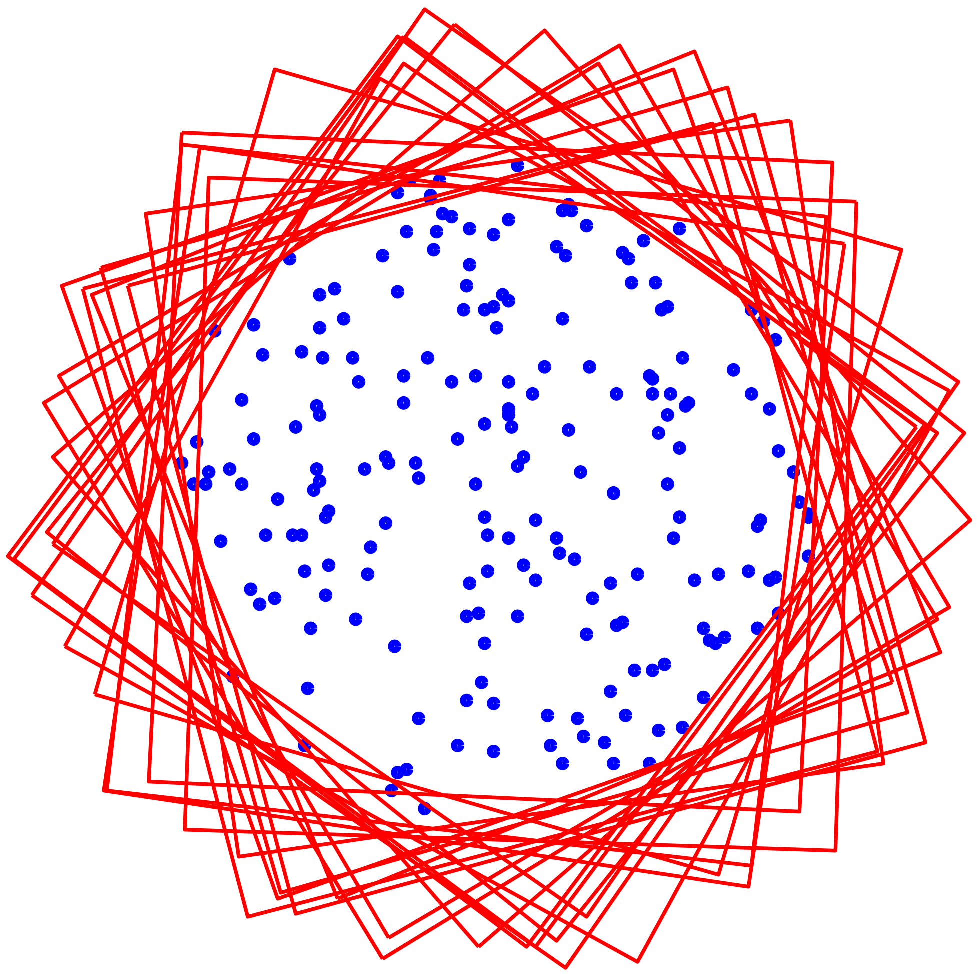
Linearisation gives a system of 2MN equations for 5M + 6N parameters ($\boldsymbol{\theta}$):

$\boldsymbol{J} \times \boldsymbol{\Delta\theta}$ = obs − calc, with Jacobian $\boldsymbol{J} = [\partial(\text{calc})/\partial\theta]$

rank($\boldsymbol{J}$) < 5M + 6N ➔ solution is not unique

What is the rank, and what does it mean?

Toy model: Numerical simulation



Numerical simulation with

M = 200 stars

N = 20 exposures

randomly distributed over 2 years

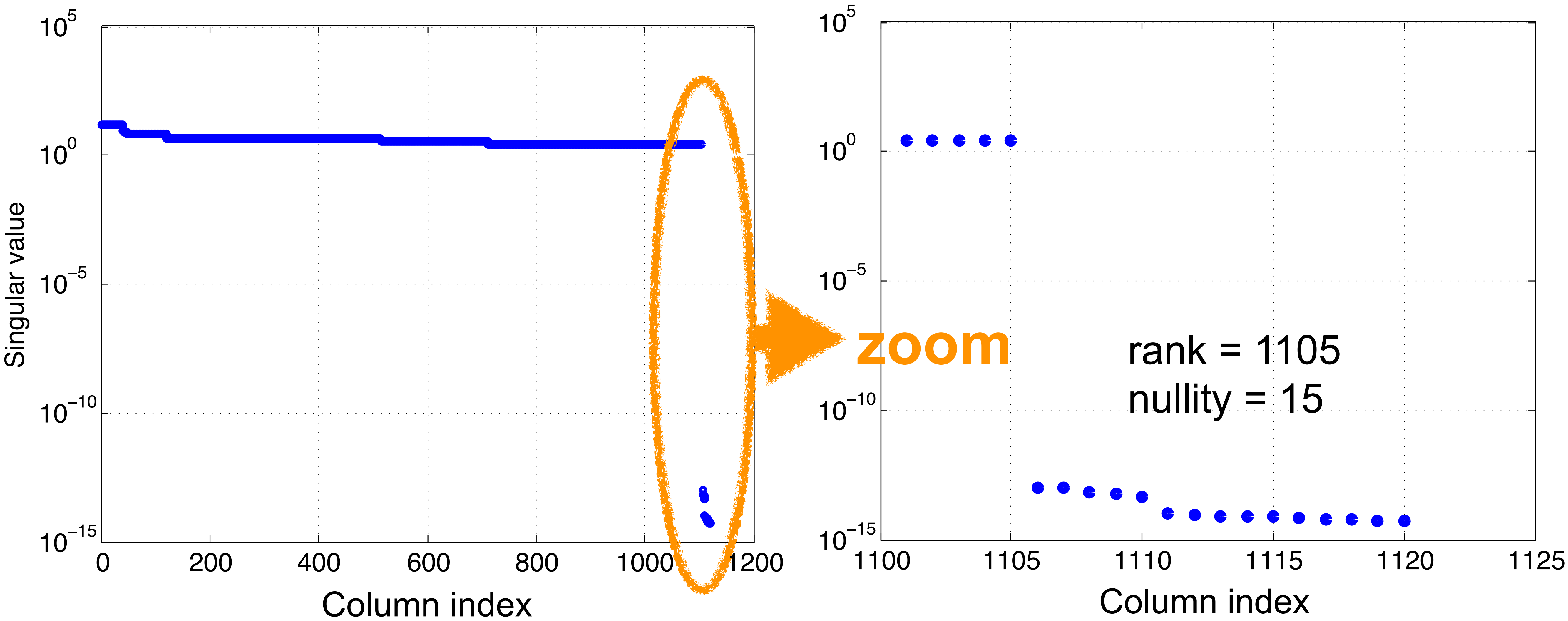
➔ 8000 equations

1120 parameters

Compute ***J*** and make SVD

(Singular Value Decomposition)

Toy model: Singular values of *J* (with 1120 parameters)



Toy model: Interpretation

Nullity = 15 ➔ the solution has 15 degrees of freedom (degeneracies)

Assume $\begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{n} \end{bmatrix}$ is a least-squares fit of the models to the data ($\boldsymbol{s} \in S_f$) .

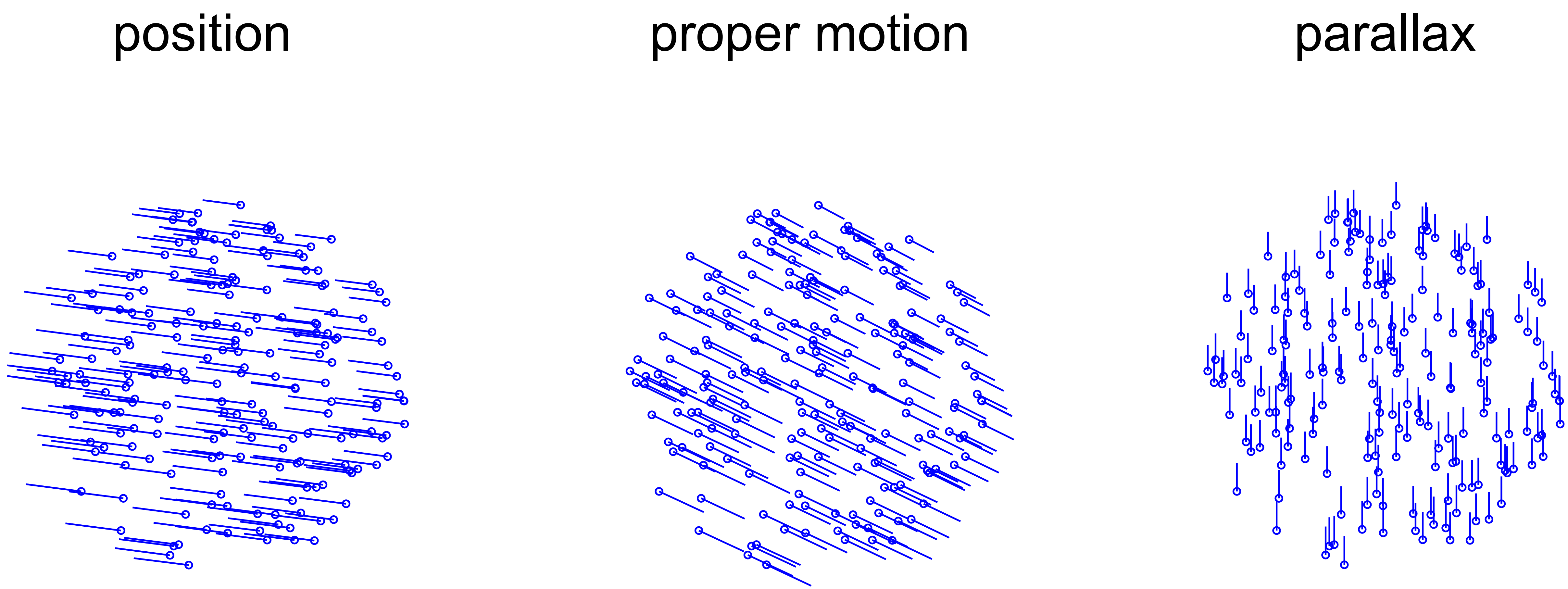
Then $\begin{bmatrix} \boldsymbol{s} + \Delta\boldsymbol{s} \\ \boldsymbol{n} + \Delta\boldsymbol{n} \end{bmatrix}$ is an equally good fit, provided that $\begin{bmatrix} \Delta\boldsymbol{s} \\ \Delta\boldsymbol{n} \end{bmatrix}$ can be written

as a linear combination of the 15 singular vectors with singular values ≈ 0.

(next 15 slides)

Why 15 ?

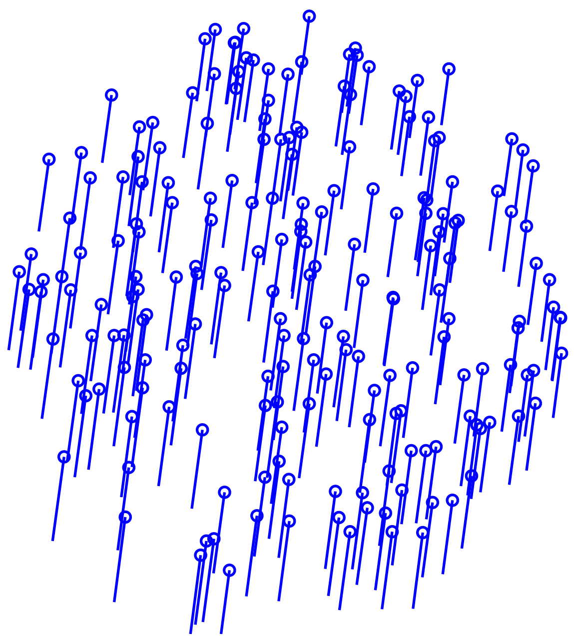
The 15 singular vectors for the toy model (# 1)



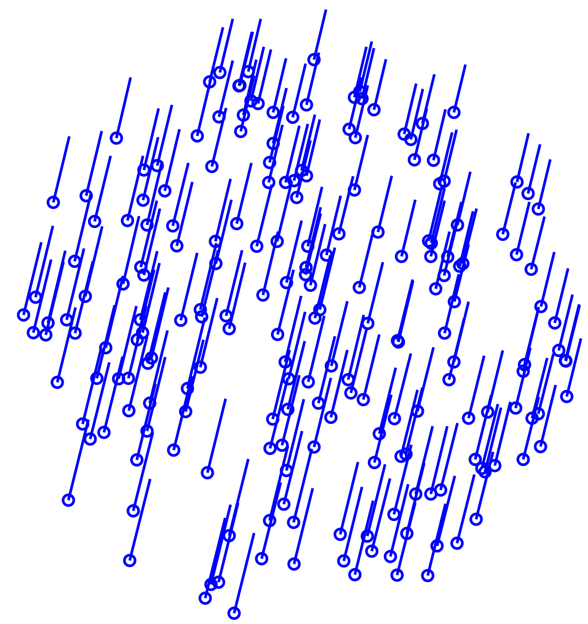
Only **Δ*s*** shown, but in each case there is an exactly "compensating" **Δ*n***

The 15 singular vectors for the toy model (# 2)

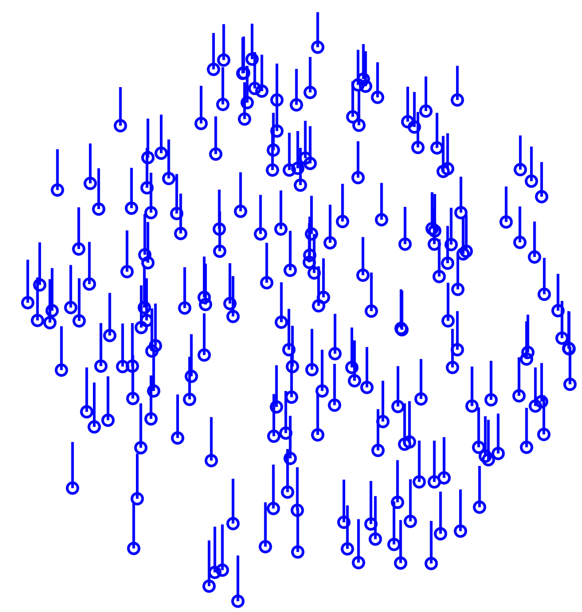
position



proper motion



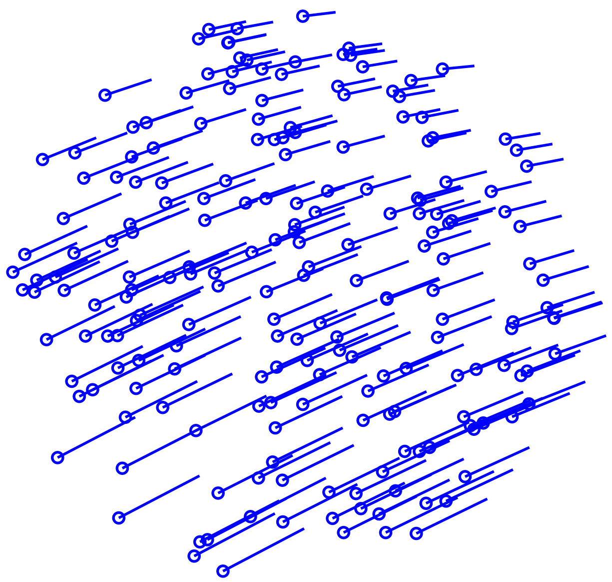
parallax



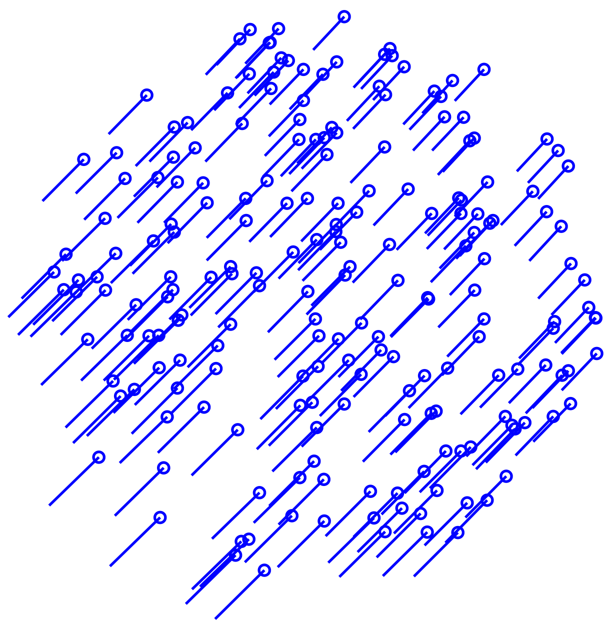
Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 3)

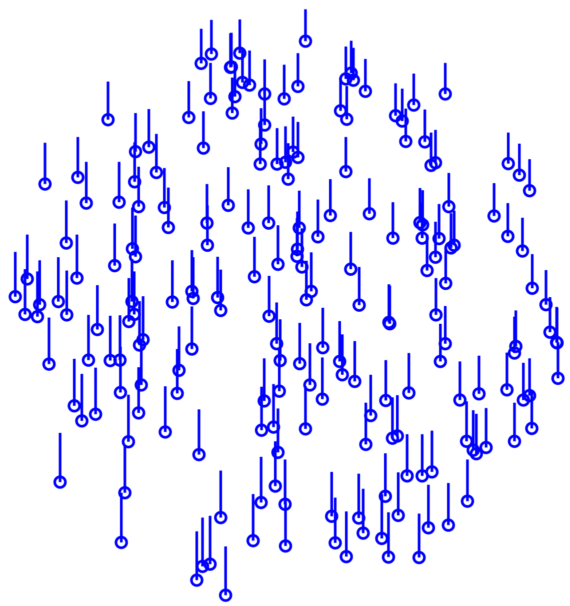
position



proper motion



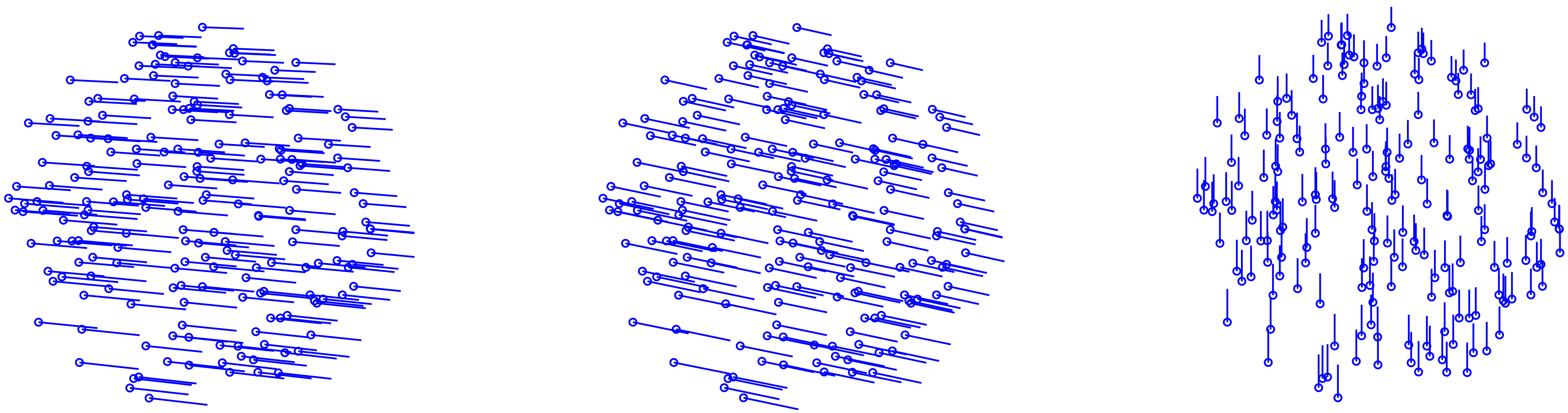
parallax



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

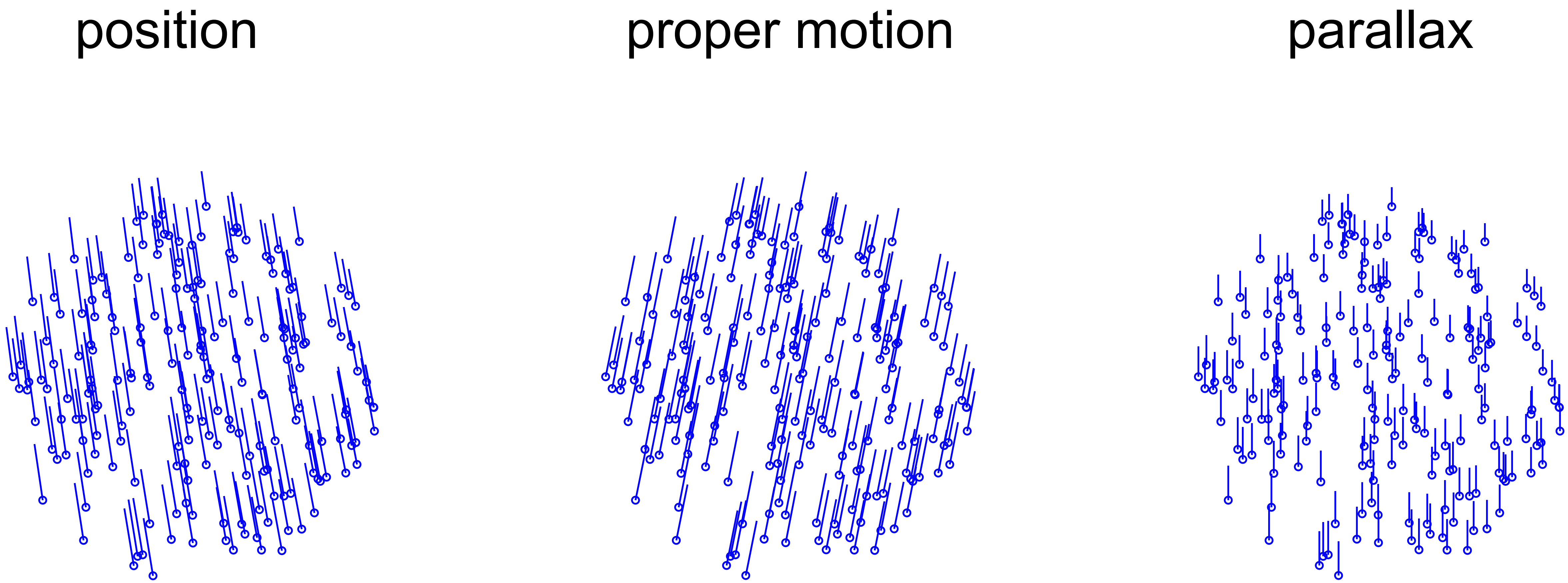
The 15 singular vectors for the toy model (# 4)

position | proper motion | parallax



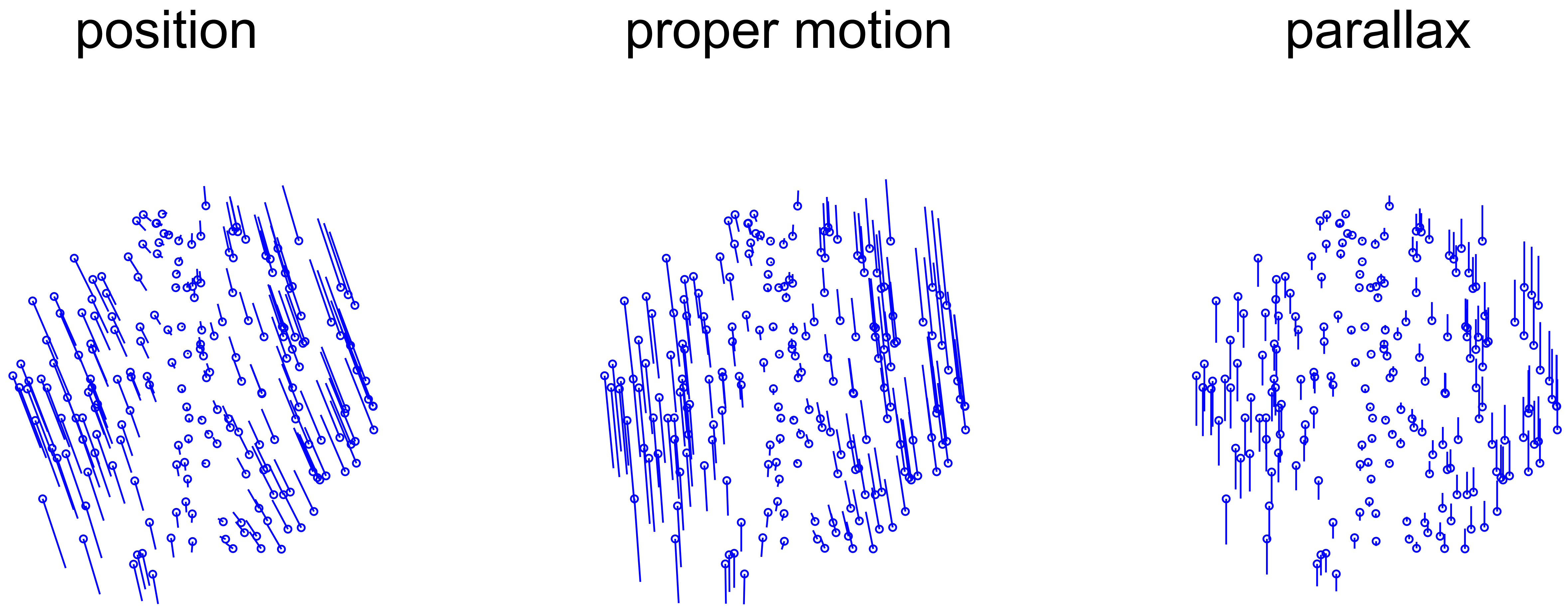
Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 5)



Only **Δ*s*** shown, but in each case there is an exactly "compensating" **Δ*n***

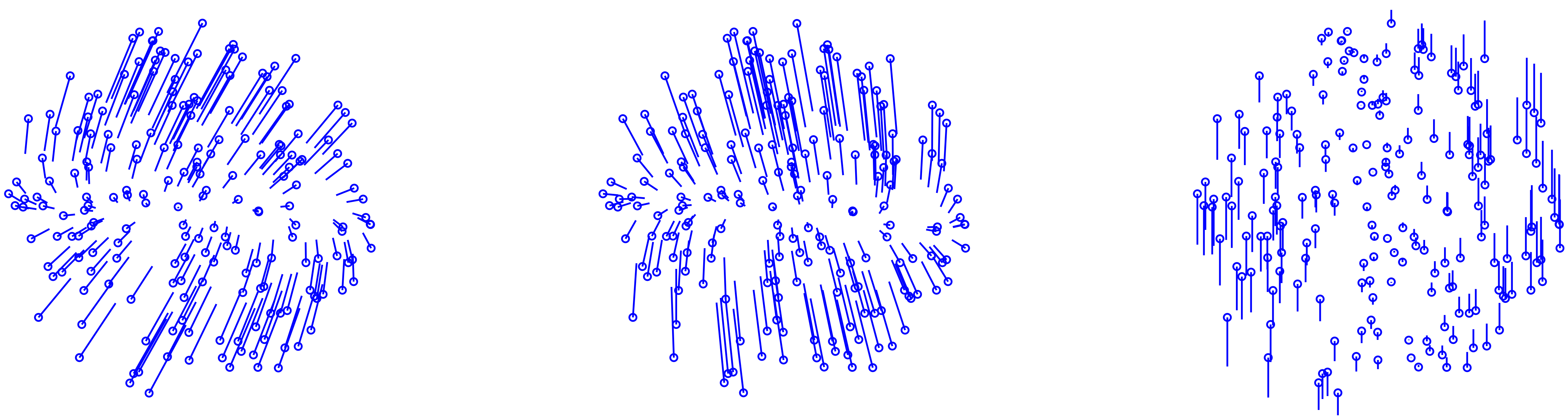
The 15 singular vectors for the toy model (# 6)



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 7)

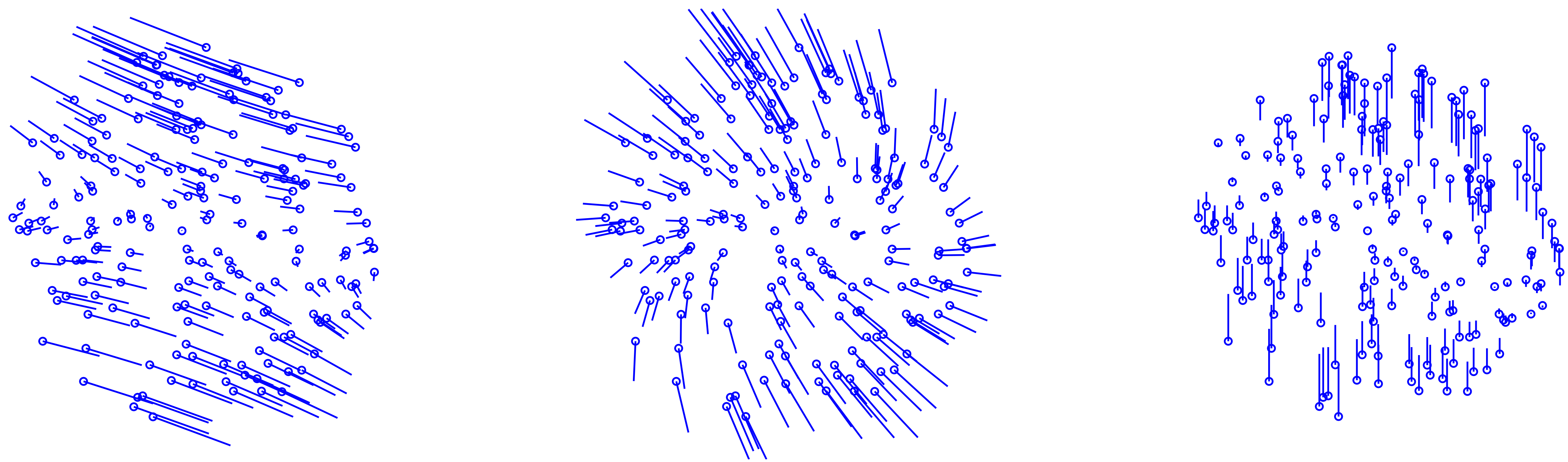
position | proper motion | parallax



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 8)

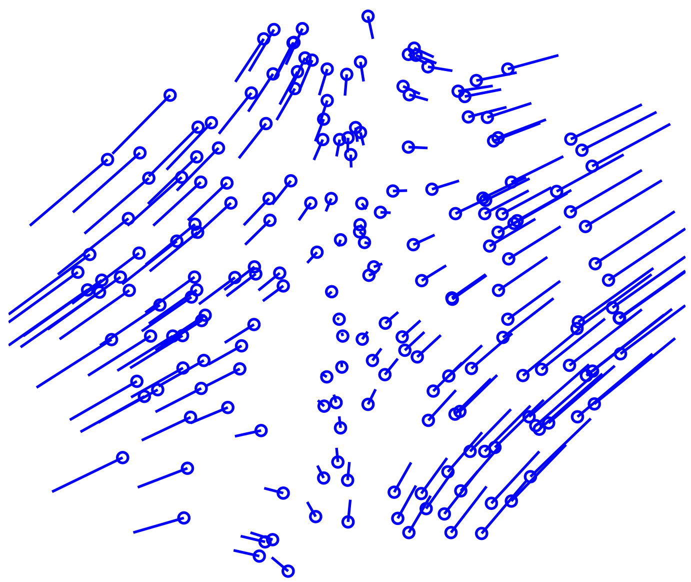
position　　　　proper motion　　　　parallax



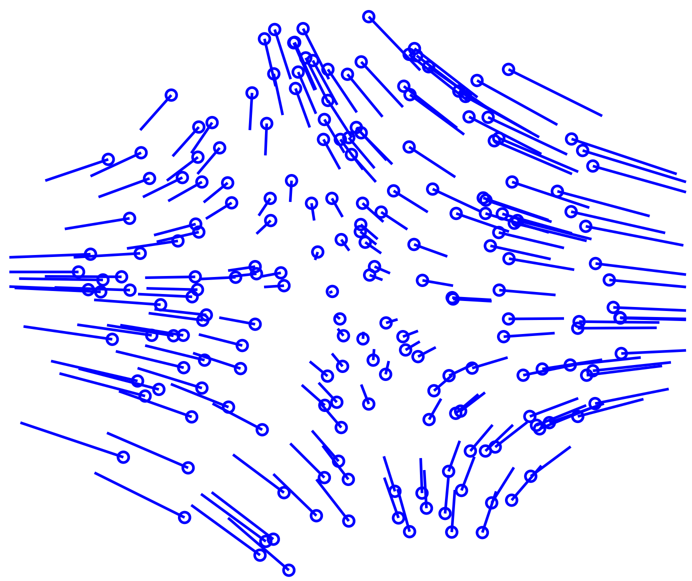
Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 9)

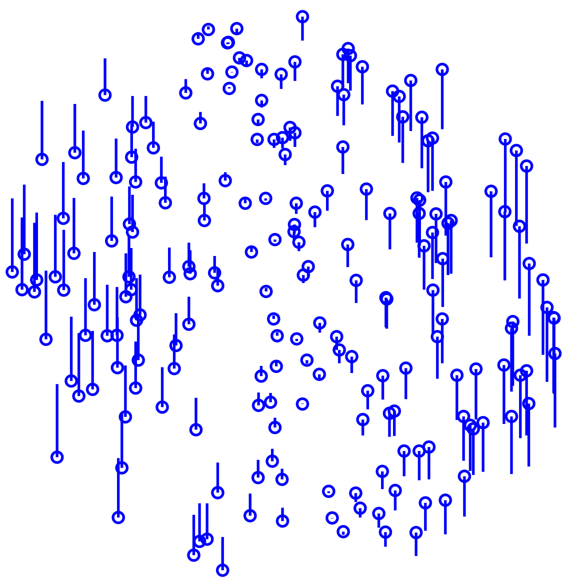
position



proper motion

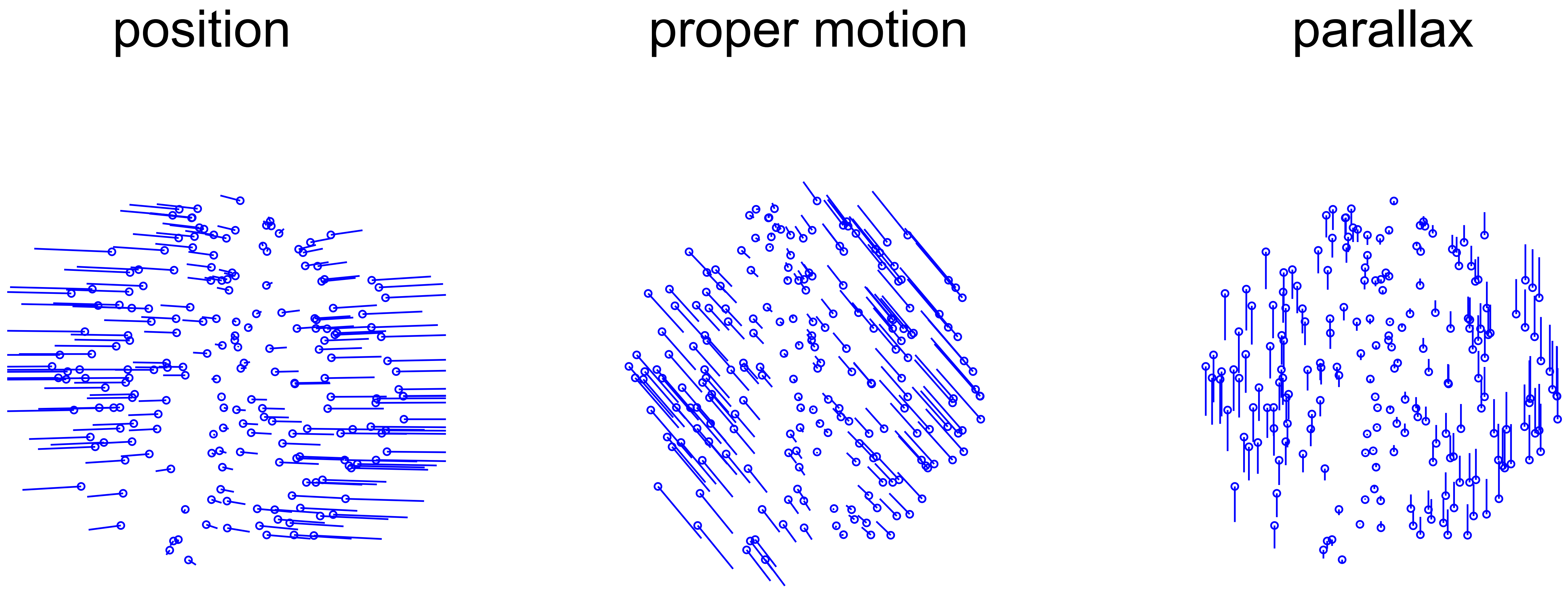


parallax



Only **Δ*s*** shown, but in each case there is an exactly "compensating" **Δ*n***

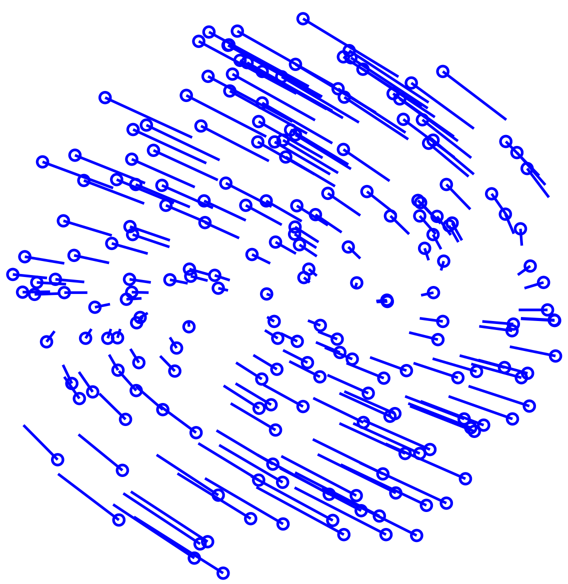
The 15 singular vectors for the toy model (# 10)



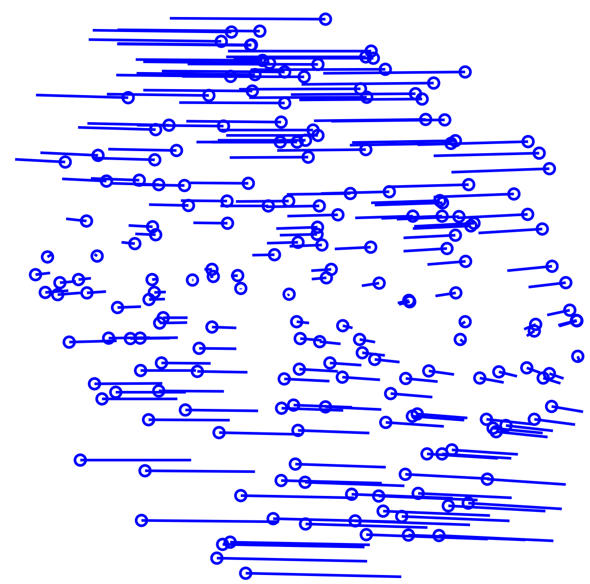
Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 11)

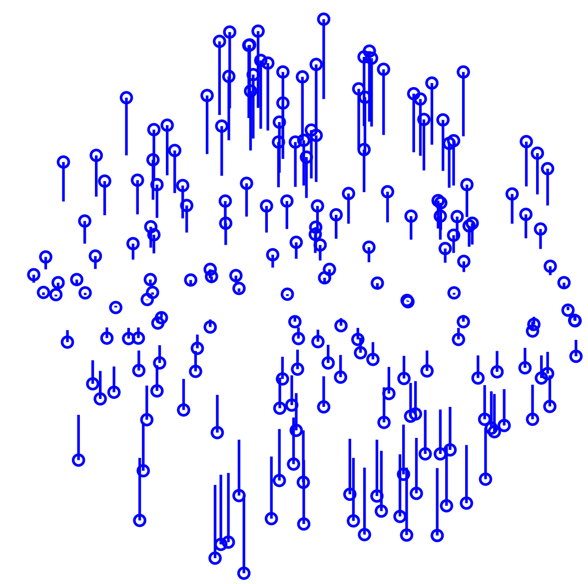
position



proper motion



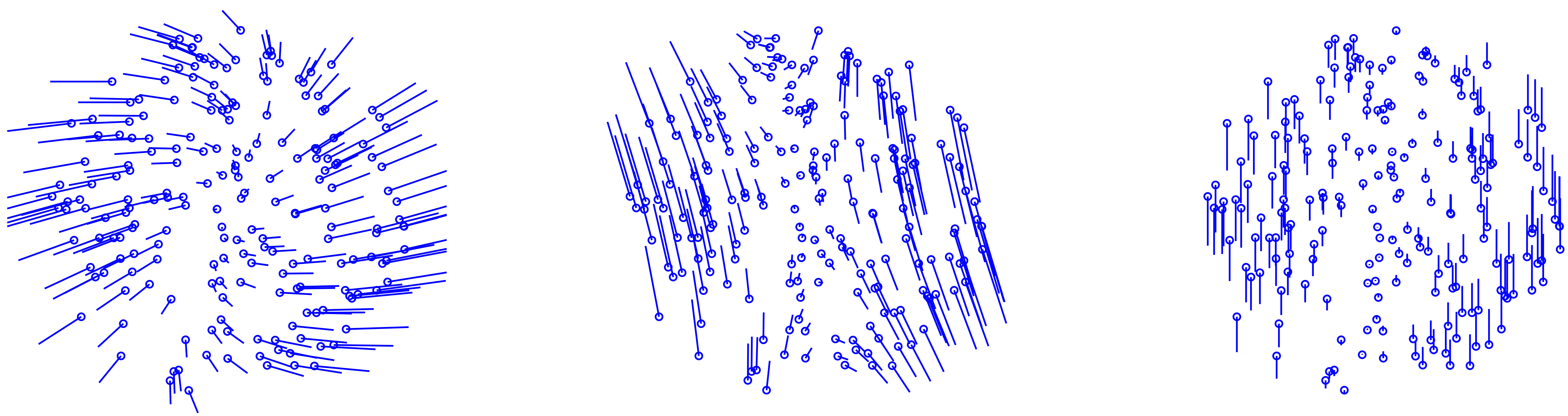
parallax



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 12)

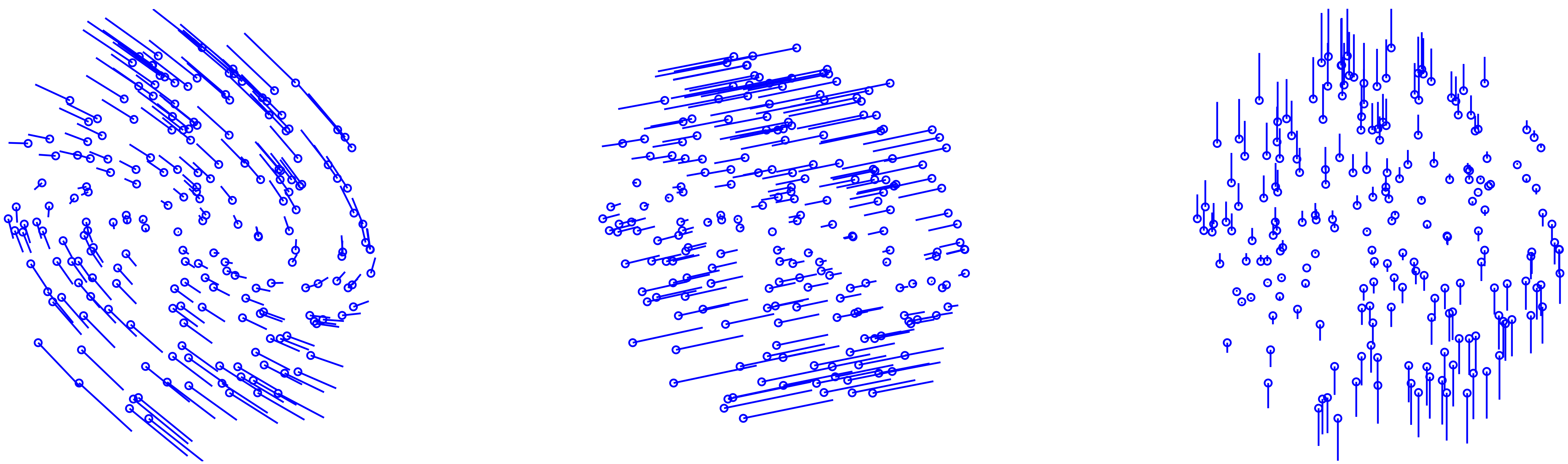
position | proper motion | parallax



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 13)

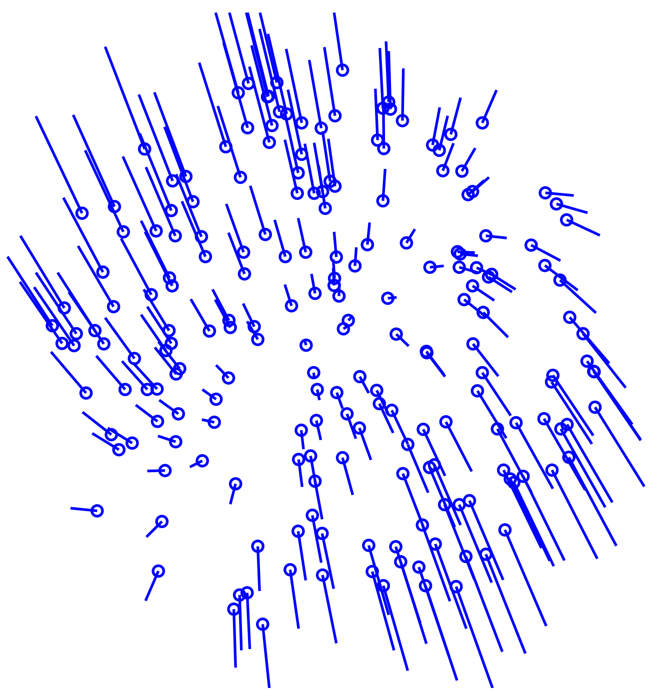
position proper motion parallax



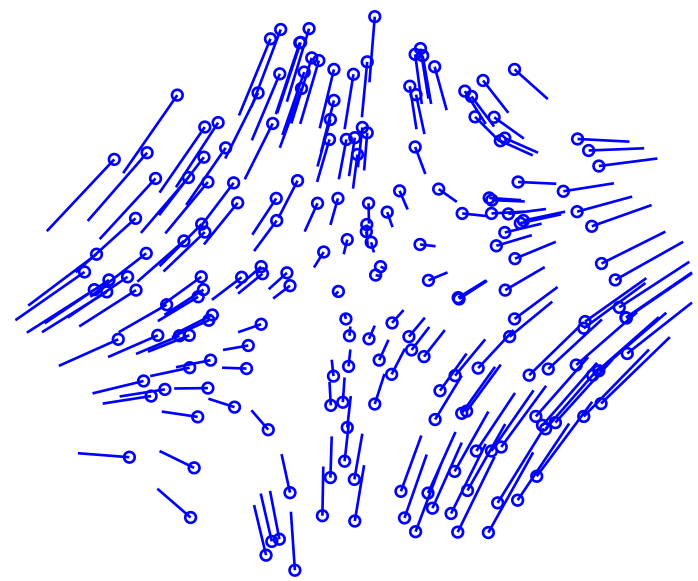
Only **Δ*s*** shown, but in each case there is an exactly "compensating" **Δ*n***

The 15 singular vectors for the toy model (# 14)

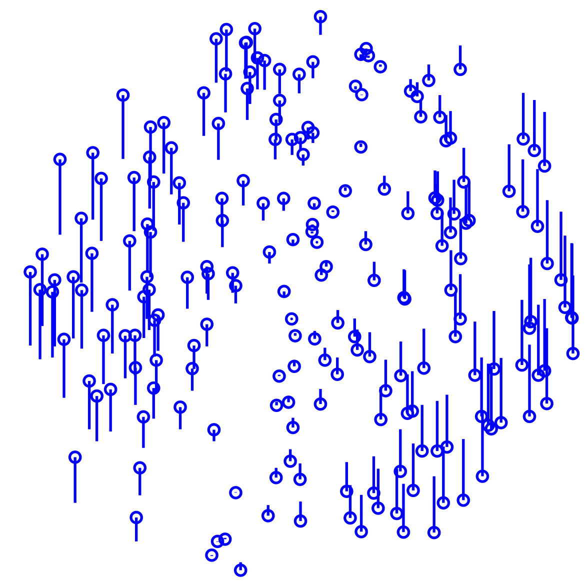
position



proper motion



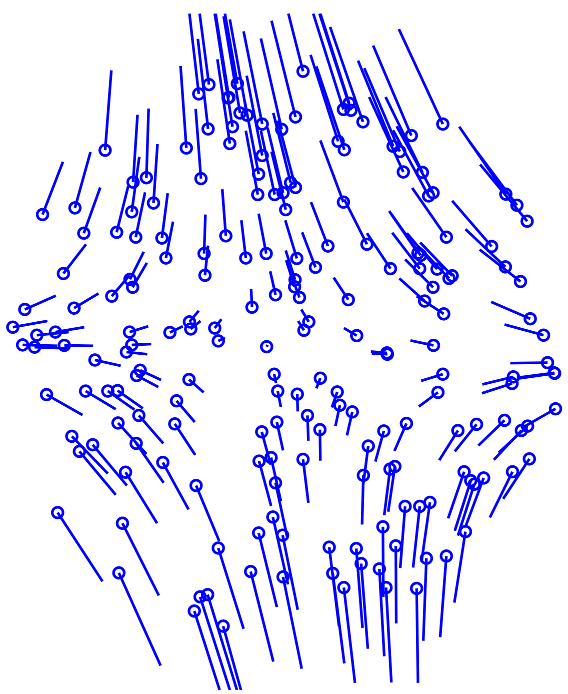
parallax



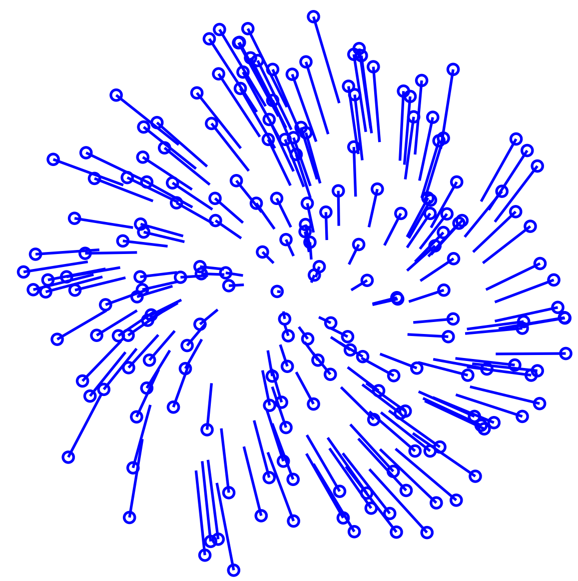
Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

The 15 singular vectors for the toy model (# 15)

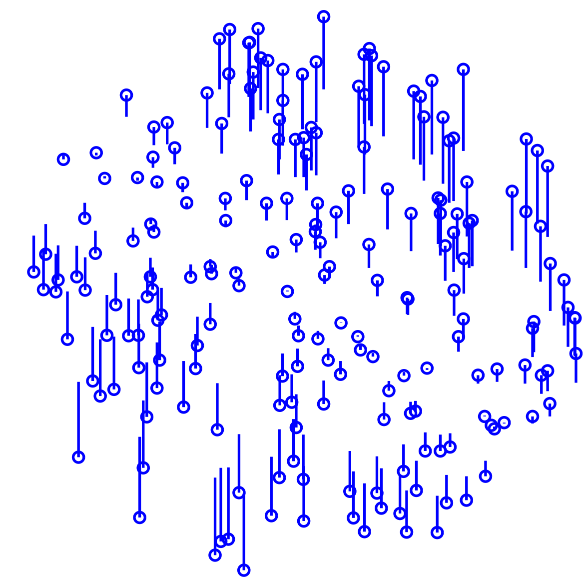
position



proper motion



parallax



Only **Δ*s*** shown, but in each case there is an exactly “compensating” **Δ*n***

Implications of the model degeneracies

Every **Δ*s*** in the solution space has a compensating **Δ*n*** (and vice versa)

Hence degeneracies -

- ***could hide actual astrophysical patterns in s***
  - the patterns are absorbed by ***n*** instead
- ***could hide actual instrumental effects in n***
  - instead, the effects become systematic errors in ***s***
- ***could be difficult to discover in complex problems***
  - in particular, none of the problems above would show up in the residuals

Dealing with the degeneracies

A few possible strategies:

1. **Accept as a practical limitation ("relative astrometry")**
   ➔ Important to know and understand the solution space

2. **Constrain the source parameters**
   ➔ E.g. use quasars for the zero point of proper motion and parallax

3. **Constrain the nuisance parameters**
   ➔ E.g. use laser metrology to fix some calibration parameters

4. **Use a different technique**
   ➔ E.g. global astrometry can eliminate many degeneracies in relative astrometry

Self-calibration for Hipparcos and Gaia

The ***Gaia astrometric global iterative solution*** uses a block-iterative method to solve

$$\min_{\boldsymbol{s},\boldsymbol{a},\boldsymbol{c}} \left\| \text{obs} - f(\boldsymbol{s},\boldsymbol{a},\boldsymbol{c}) \right\|_{\mathcal{M}}$$

− nuisance parameters are the attitude (***a***) and geometric calibration (***c***)

A similar method was used for the Hipparcos re-reduction (van Leeuwen 2007)

	Number of parameters (millions)		
	s	***a***	***c***
Hipparcos	0.5	1	0.05
Gaia DR1	10	1.5	0.1
Gaia (final)	100	5	1

(Counting only the primary solution and along-scan data)

The limits of self-calibration

- The astrometric solutions for Hip and Gaia involve ***millions*** of parameters

- Some degrees of freedom are well known and explicitly taken care of in the solutions (e.g. the reference frame)

- Can we confidently say we know and understand all the degrees of freedom?

- Numerical simulations are helpful: SVD may not be feasible, but one can generate random vectors ($\Delta\boldsymbol{s}$, $\Delta\boldsymbol{n}$) in the solution space

Conclusions

Self-calibration is great but cannot determine everything!

- ➔ For interpreting the results one needs to know the solution space S_f
- ➔ This depends on the models used (f), not on the data

Very careful attention should be given to the calibration models in complex projects such as Gaia

- ➔ Unrecognised degrees of freedom could produce systematics that are not revealed by the residuals
- ➔ Numerical simulations may be the only practical way to explore possible weaknesses in the solution