Relativistic Models for Gaia and beyond

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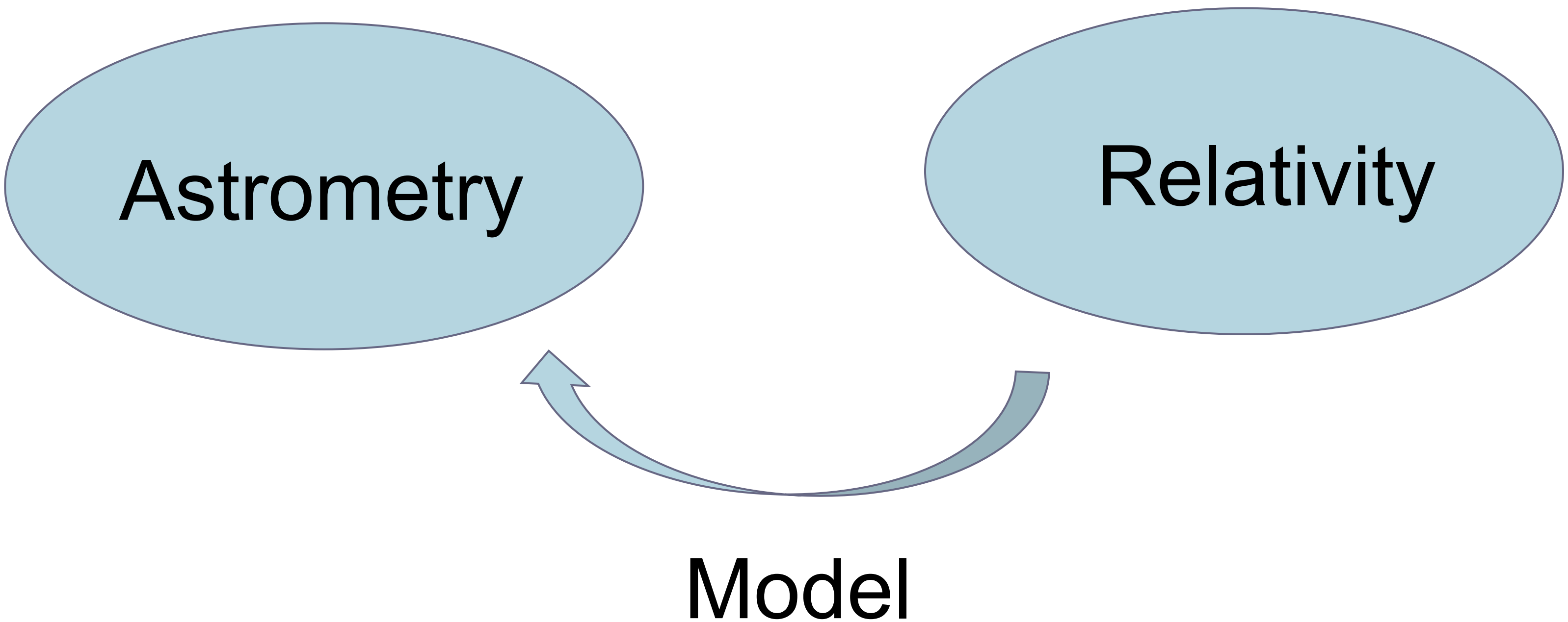

Astrometry
Relativity
Model

Gaia Relativity Model (GREM)

- Standard IAU relativistic reference systems (Soffel et al. 2003) form the basis for the Gaia data processing

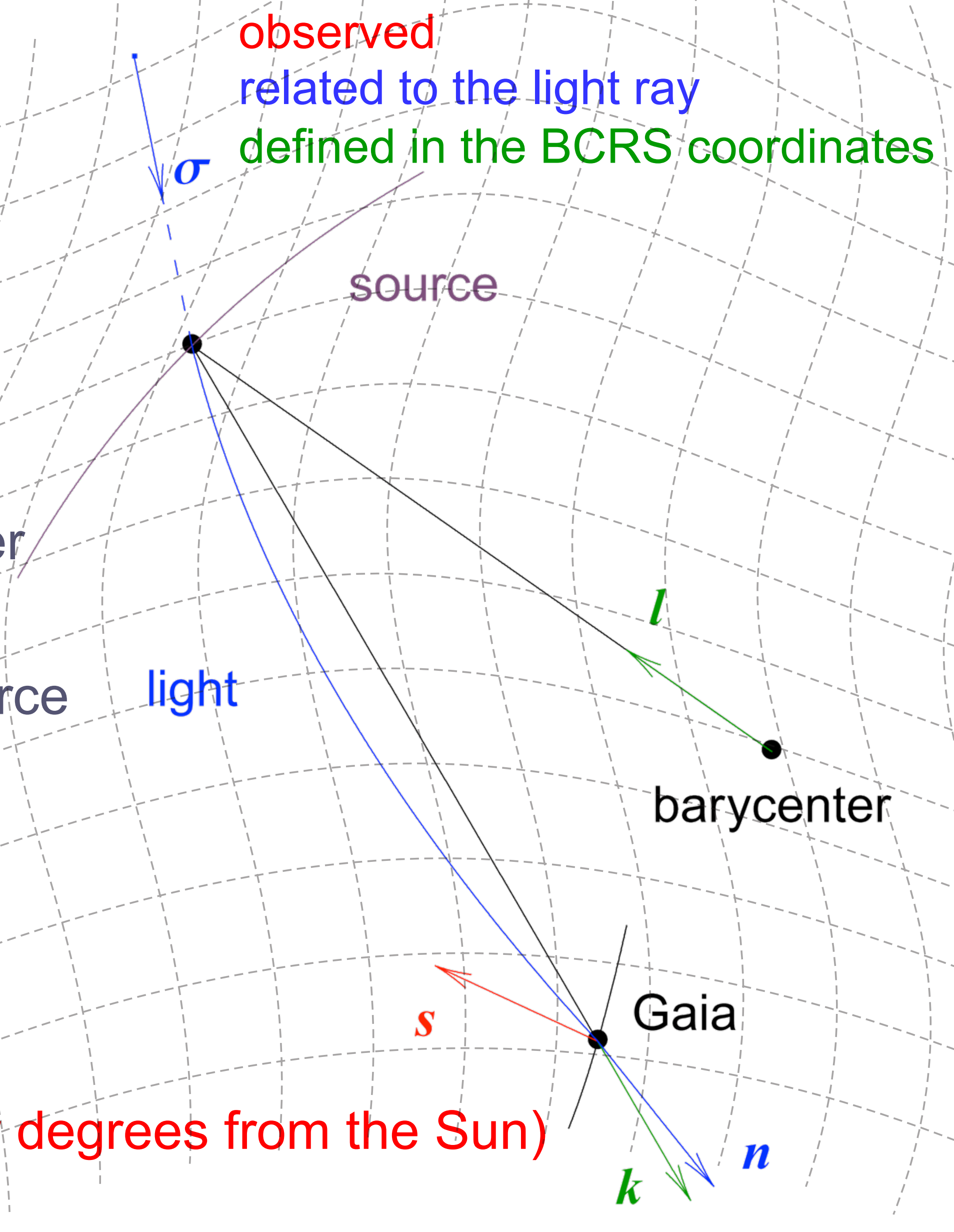
- Relativistic model for astrometric observations (Klioner 2003, 2004):
  - aberration via Lorentz transformations
  - deflection of light: monopole (post- und post-post-Newtonian), quadrupole and gravitomagnetic terms up to 17 bodies routinely, more if needed
  - relativistic definitions of parallax, proper motion, etc.
  - relativistic definitions of observables and the attitude of the satellite
  - relativistic model for the synchronization of the Gaia atomic clock and ground-based time scale (Gaia proper time etc.)

Consistency of all aspects of the modeling (constants, ephemerides, etc.) should be ensured and monitored.

Accuracy: 0.1 µas (at a distance larger than a few degrees from the Sun)

Gaia Relativity Model (GREM)

- **s** the observed direction
- **n** tangential to the light ray at the moment of observation
- **σ** tangential to the light ray at $t = -\infty$
- **k** the coordinate direction from the source to the observer
- **l** the coordinate direction from the barycentre to the source
- ϖ the parallax of the source in the BCRS



Accuracy limit: 0.1 μas
(at a distance larger than a few degrees from the Sun)

A better model?

I. Modeling of the light propagation in the gravitational field of solar system

Only needed if considerably higher accuracy is considered (<0.1 μas)

- Theory:
  - light propagation in the field of N arbitrarily moving bodies with arbitrary multipole structure to second order in G

    Even the metric tensor is currently unknown!
  - some progress over recent years...
- Practice: more accurate data for the model
  - barycentric velocity and position of the orbit (e.g. velocity to <0.1 mm/s)
  - better solar system ephemerides (e.g. positions to <1 km; masses)

Some options (to live without a full-scale model):

1) use numerical computations instead of analytical formulas
2) avoid difficult cases
3) determine the critical parameters from astrometry itself

Auxiliary data: light deflection due to minor bodies

- A (spherical) body of mean density ρ produces a light deflection not less than δ if its radius:

$$R \geq \left(\frac{\rho}{1\ \mathrm{g/cm^3}} \right)^{-1/2} \times \left(\frac{\delta}{1\ \mu\mathrm{as}} \right)^{1/2} \times 650\ \mathrm{km}$$

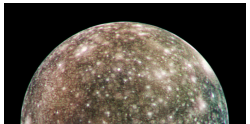
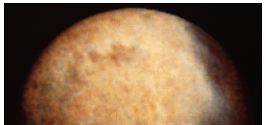
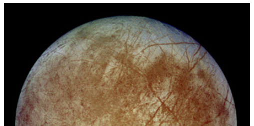
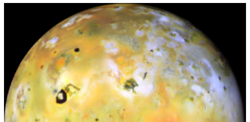
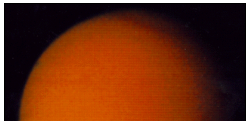
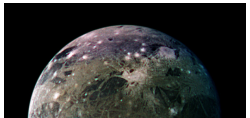
$$\begin{matrix} \delta = 1\ \mathrm{nas} \\ \rho \leq 3\ \mathrm{g/cm^3} \end{matrix} \quad \Rightarrow \quad R \geq 10\ \mathrm{km}$$

Not too bad!

1. By the time when the problem becomes practical we will know all these bodies in the Solar system…

2. Assuming relativity is correct one can simply fit the mass of the bodies: e.g. Gaia is able to determine the mass of Jupiter to about 10^{-3}

3. Million(s) of deflecting bodies? Big computers are needed?

A better model?

II. Better modeling of source trajectory

- Hipparcos, Gaia: linear motion in space treated exactly
  (in some relativistic coordinates!)

  Correct for a good fraction of sources – enough sources for a solution

- for a higher accuracy (separately or through a combination with Gaia):

  - accelerated motion in space
  - light travel time effects (Butkevich, Lindegren, 2014)

  No need to have closed-form analytical formulas.
  Numerical algorithms can be used: computers only get faster!

Fundamental limits for the accuracy?

Noise in the light propagation that cannot be modeled:

- (unknown) minor bodies in solar system (possibly, not a problem)
- unknown massive objects in the Galaxy (the biggest problem?)
- stochastic gravitational wave background (it certainly exists at some level and does represent a fundamental limit of accuracy!)

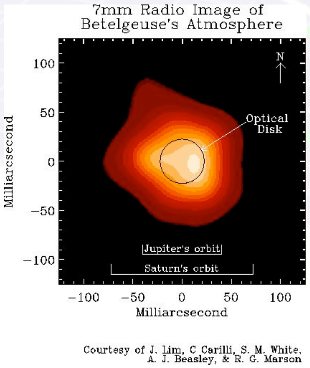
The level of these effects is only partially known.

Non-relativistic limitation:

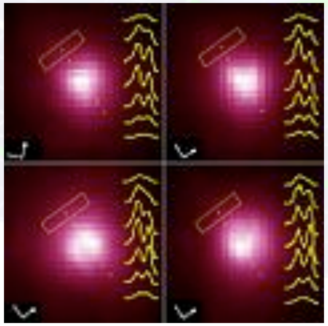
source structure!

Do we have enough point-like sources at higher accuracies?

Betelgeuse

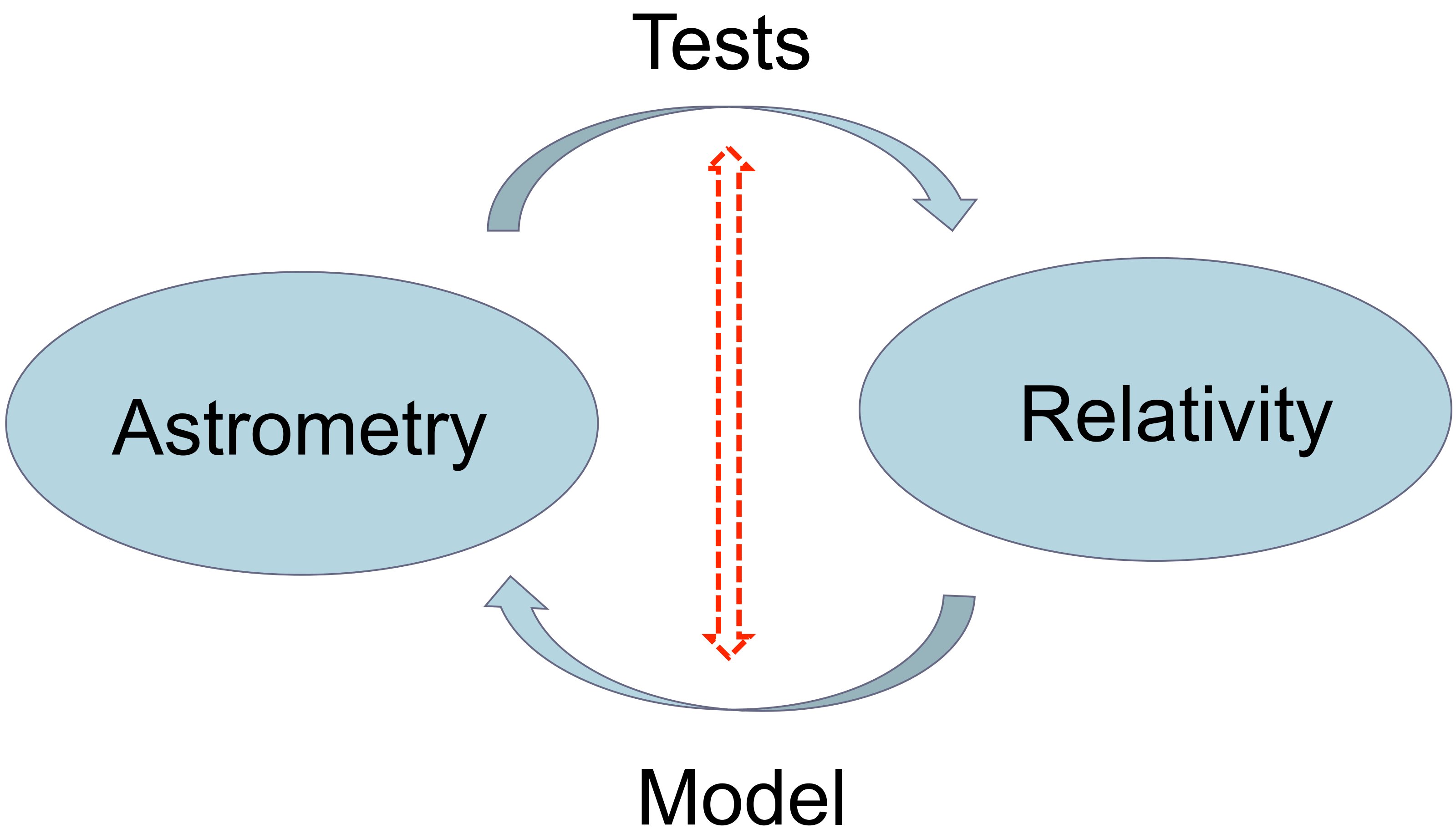


Radio 7mm



UV-pulsations, HST

VLT-NACO



Each relativistic effect used in the models can be used to test GR

Possible tests: another attempt of classification

I. Non-gravitational tests: Local Lorentz Invariance (special relativity)

II. Weak-field tests in the solar system:
- all sorts of the deflection of light
- motion of solar system bodies

III. Observations of remote objects in strong-field regime
- Compact binaries
- Stars at the Galactic center (close to the SMBH)

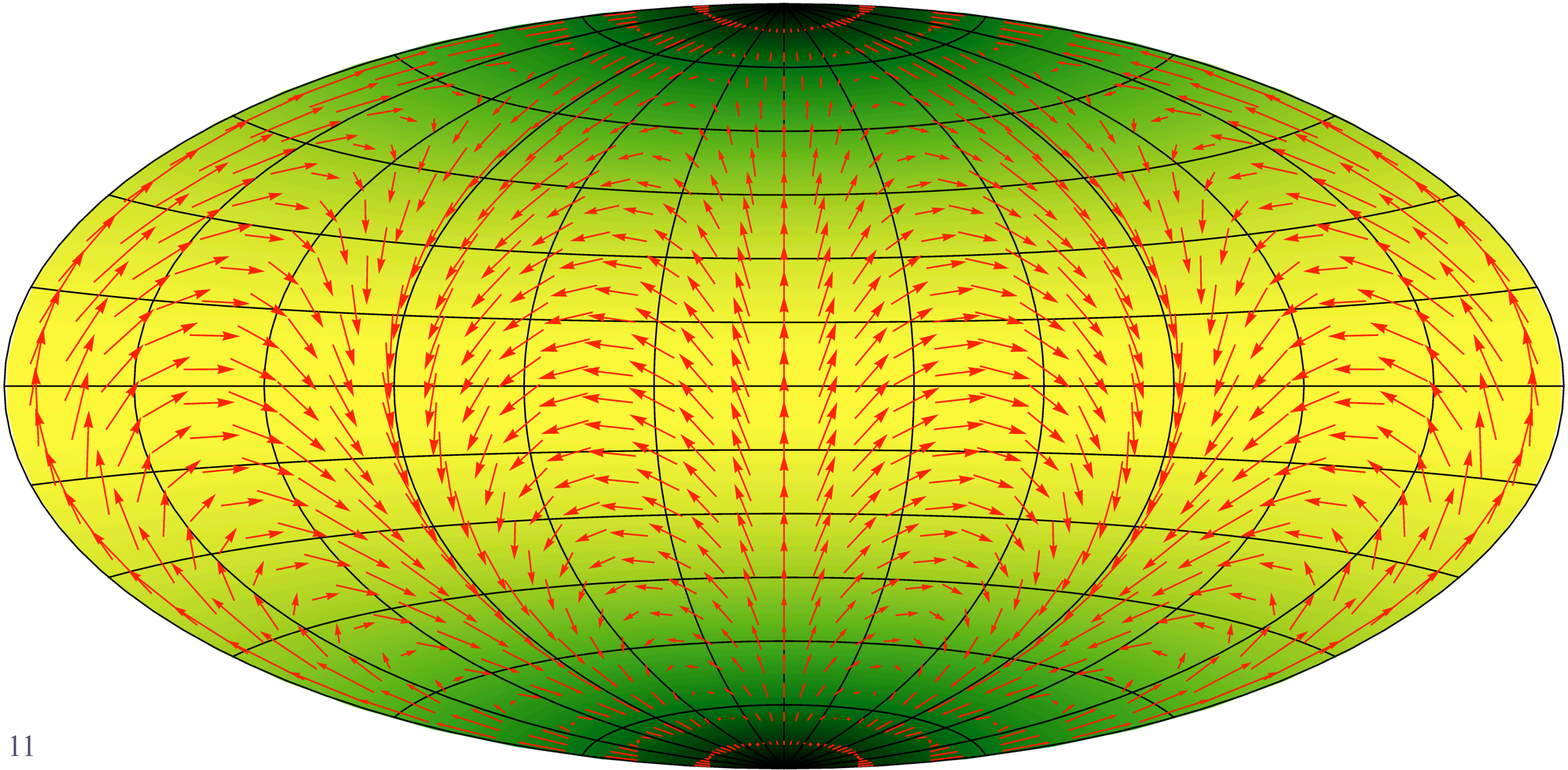
IV. Cosmological tests from highly accurate proper motions

V. Astrometry as a gravitational wave detector
- Ultra low frequency, primordial gravitational waves
- Higher frequency gravitational waves from binary supermassive black holes

Gravitational waves and astrometry

- At each moment of time a GW produces a deflection pattern on the sky: it is not a pure quadrupole, but rather close to it (Braginsky et al, 1991; Pyne et al, 2006; Gwinn et al, 2006; Book, Flanagan, 2011; Klioner, 2014-)

This is for a GW propagating in the direction δ=90° ("+" polarization)



Plane gravitational waves and astrometry

- Deflection of light coming from stars with distances $r \gg \lambda_{\rm gw}$

Direction towards the astrometric source (star): $\mathbf{u}$

$$\delta u^i = \frac{u^i + p^i}{2(1+\mathbf{u}\cdot\mathbf{p})} h_{jk} u^j u^k - \frac{1}{2} h_{ij} u^j$$

Direction of propagation of the gravitational wave: $\mathbf{p}$

Metric perturbation at the observer located at $\mathbf{x}_{\rm obs}$

$$h_{ij} = h^+ p^+_{ij} \cos\left(2\pi\nu\left(t - \frac{1}{c}\mathbf{p}\cdot\mathbf{x}_{\rm obs} - t^+\right)\right)$$

$$+ h^\times p^\times_{ij} \cos\left(2\pi\nu\left(t - \frac{1}{c}\mathbf{p}\cdot\mathbf{x}_{\rm obs} - t^\times\right)\right)$$

Two polarizations:

$$p^+_{ij} = \left(\mathbf{P}\,\mathbf{e}^+\,\mathbf{P}^{\rm T}\right)_{ij}$$

$$p^\times_{ij} = \left(\mathbf{P}\,\mathbf{e}^\times\,\mathbf{P}^{\rm T}\right)_{ij}$$

Frequency: ν

Phases: $t^+,\ t^\times$

Strains: $h^+,\ h^\times$

This is only a part of the signal: the source-term is ignored => noise

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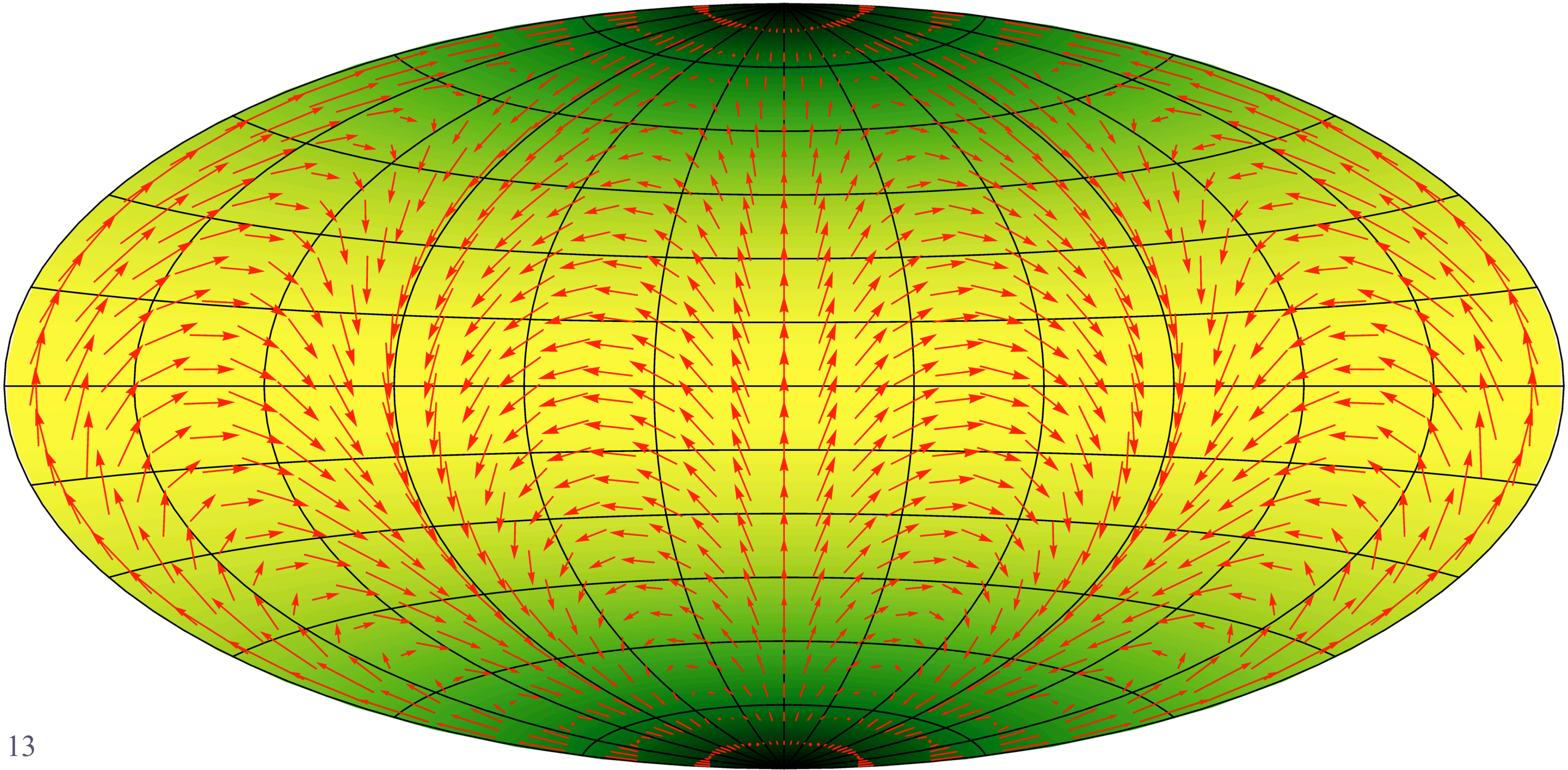
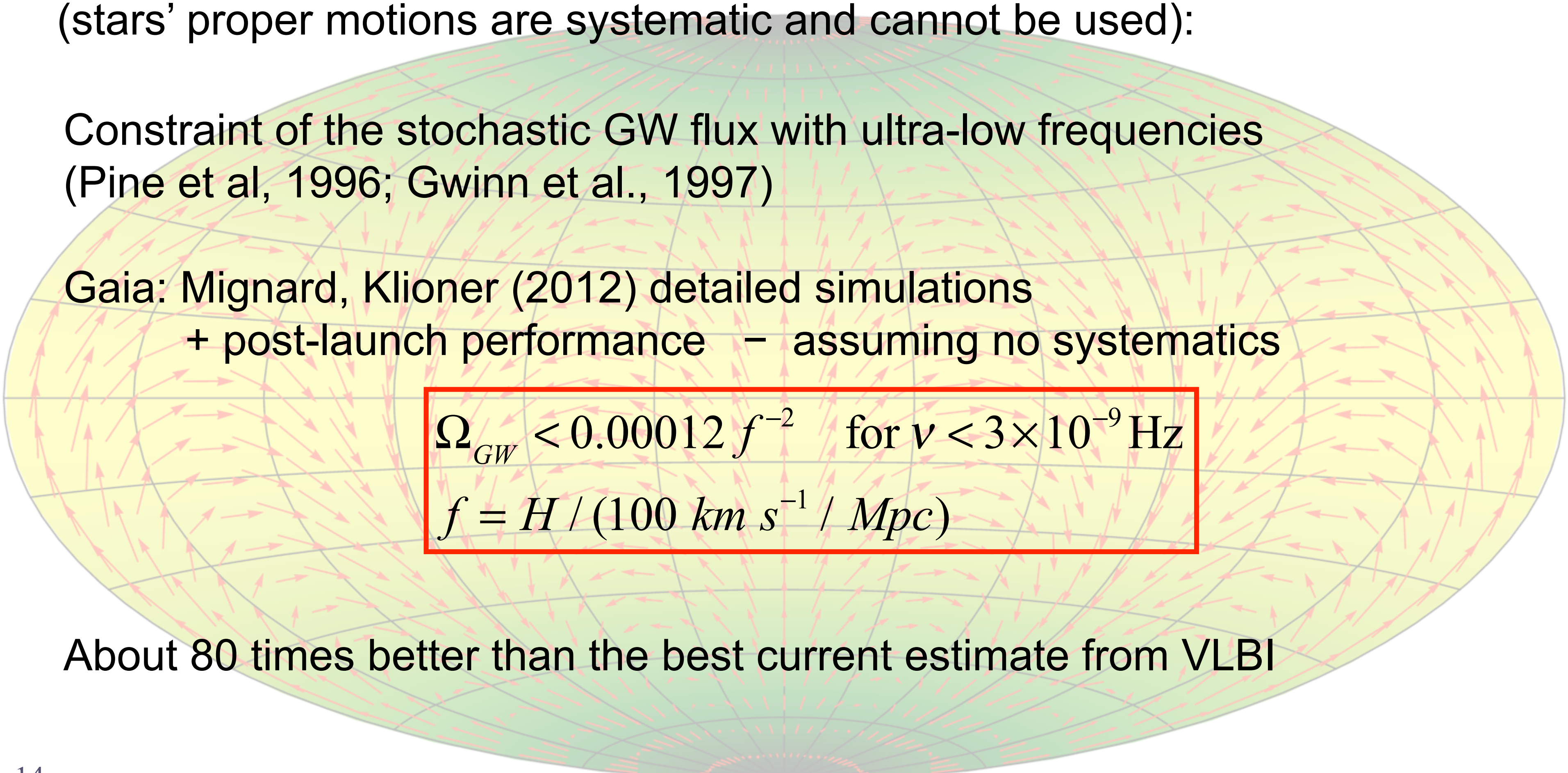


Application 1: ultra-low-frequency GWs

If the frequency of the GW is so small that the period of the wave is substantially larger than the time span covered with observations, the GW deflection pattern is absorbed by proper motion parameters.

This is now the pattern in the proper motions of QSOs in the final catalogue (stars' proper motions are systematic and cannot be used):



Constraint of the stochastic GW flux with ultra-low frequencies (Pine et al, 1996; Gwinn et al., 1997)

Gaia: Mignard, Klioner (2012) detailed simulations
+ post-launch performance − assuming no systematics

$$\Omega_{GW} < 0.00012\, f^{-2} \quad \text{for } \nu < 3\times 10^{-9}\ \text{Hz}$$

$$f = H / (100\ km\ s^{-1} / Mpc)$$

About 80 times better than the best current estimate from VLBI

Gravitational Wave Spectrum

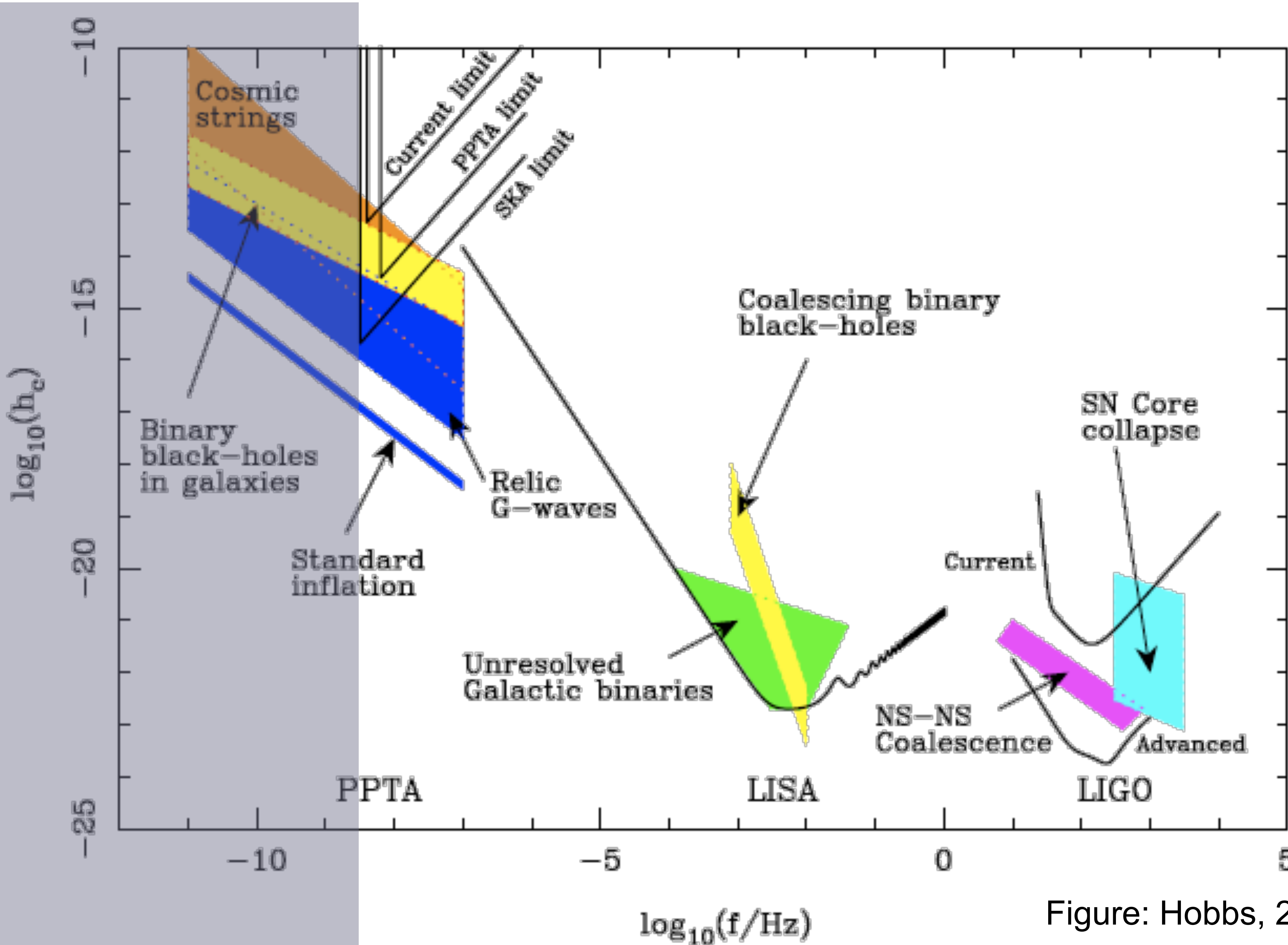
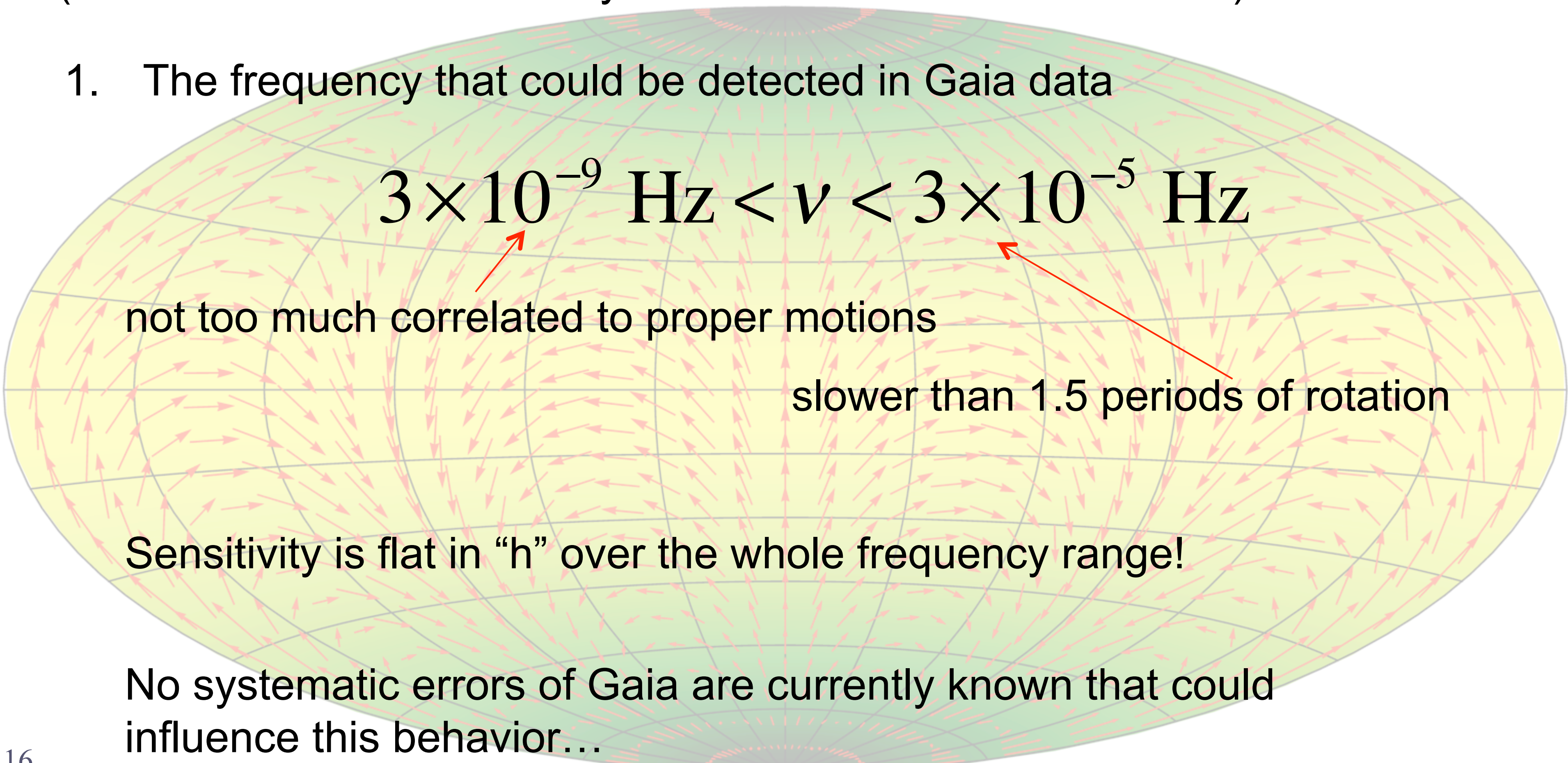


Figure: Hobbs, 2008

Application 2: low-frequency GWs

If the frequency of the GW is large enough, the time-dependence of the deflection does not allow the effect to be absorbed by proper motion.

This is now a time-dependent pattern in the residuals of the solution (at each moment of time only certain directions are observed):



1. The frequency that could be detected in Gaia data

$$3\times10^{-9}\ \mathrm{Hz} < \nu < 3\times10^{-5}\ \mathrm{Hz}$$

not too much correlated to proper motions

slower than 1.5 periods of rotation

Sensitivity is flat in “h” over the whole frequency range!

No systematic errors of Gaia are currently known that could influence this behavior…

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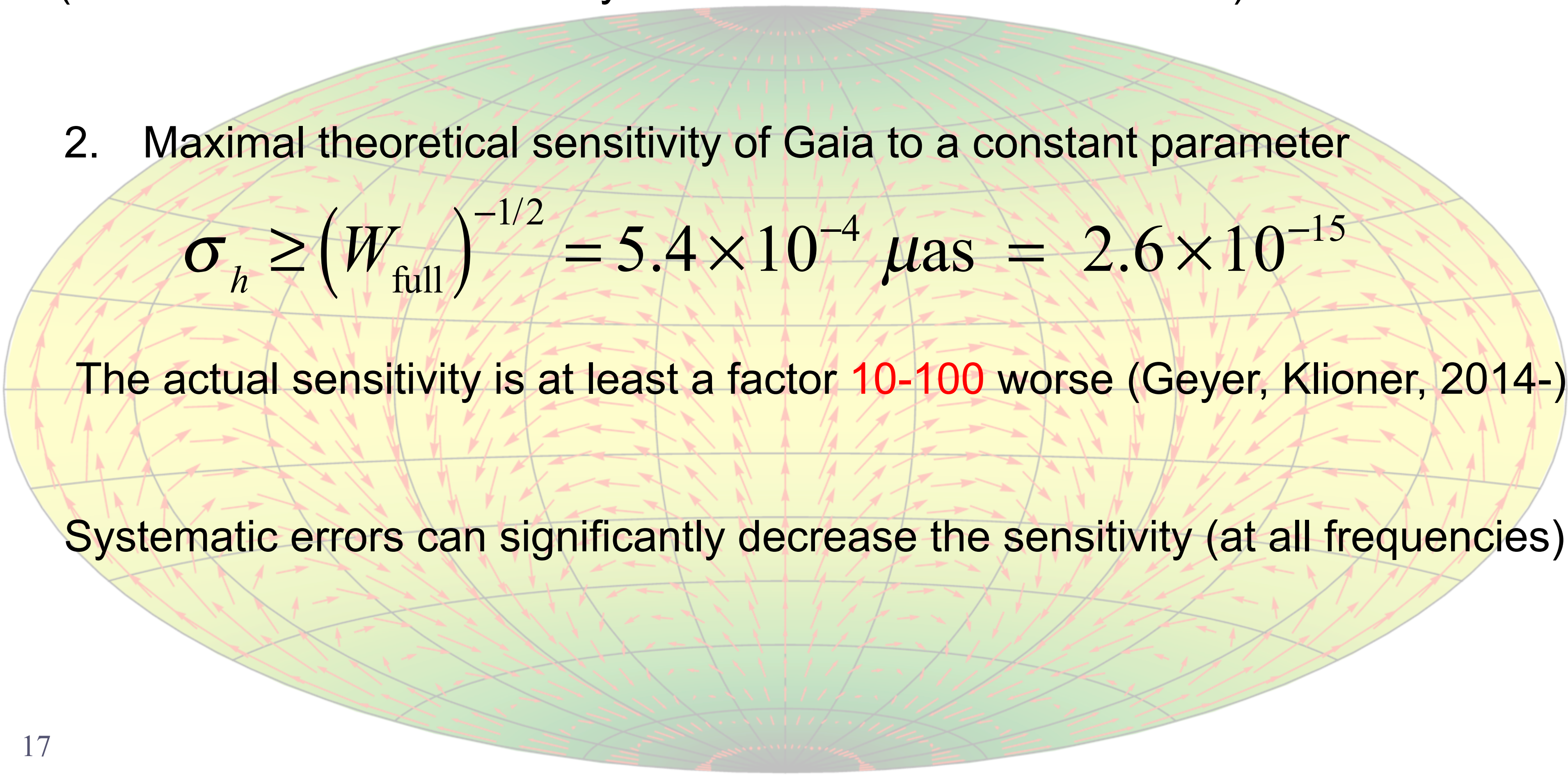


2. Maximal theoretical sensitivity of Gaia to a constant parameter

$$\sigma_h \geq \left(W_{\rm full}\right)^{-1/2} = 5.4\times10^{-4}\ \mu{\rm as} \ = \ 2.6\times10^{-15}$$

The actual sensitivity is at least a factor 10-100 worse (Geyer, Klioner, 2014-)

Systematic errors can significantly decrease the sensitivity (at all frequencies)

Low-frequency GWs: complications

1. Only two spots on the sky are observed at a given moment
2. Observations are essentially one dimensional: “along scan”
3. Astrometric parameters are fitted from the same data:
   standard astrometric solution absorbs a part of the GW signal
   (Geyer, Klioner, 2017)
4. Highly non-linear optimization problem:

   search in GW frequency is needed

5. Huge amount of data: more than 10^{12} observations

   data compression is needed: normal points,
   sky pixelization,
   vector spherical harmonic analysis

6. Systematic errors of the instrument

Sources of low-frequency continuous GWs

Binary SMBHs: a binary system with a chirp mass $\mathcal{M}$ on a circular orbit
with orbital period $2P_{\rm gw} \gg 20^{h}\mathcal{M} \;/\,(10^{9}\;M_{\odot})$
at luminosity distance r

$$h = \frac{4\pi^{2/3}}{c^4}\left(G\mathcal{M}\right)^{5/3} P_{\rm gw}^{-2/3} r^{-1} = 1.19\times10^{-14}\left(\frac{\mathcal{M}}{10^9 M_\odot}\right)^{5/3}\left(\frac{P_{\rm gw}}{1\ \rm yr}\right)^{-2/3}\left(\frac{r}{100\ \rm Mpc}\right)^{-1}$$

Frequency drift due to energy loss

$$\dot{f}_{\rm gw} = \frac{96}{5}\pi^{8/3}\left(\frac{GM}{c^3}\right)^{5/3} f_{\rm gw}^{11/3} = 5.83\times10^{-12}\ \rm Hz/yr\left(\frac{M}{10^9 M_\odot}\right)^{5/3}\left(\frac{P_{\rm gw}}{1\ \rm yr}\right)^{-11/3}$$

Time to coalescence

$$\tau = \frac{3}{8}\frac{f_{\rm gw}}{\dot{f}_{\rm gw}} = \frac{5}{256}\pi^{-8/3}\left(\frac{GM}{c^3}\right)^{-5/3} P_{\rm gw}^{8/3} = 2039\ \rm yr\left(\frac{M}{10^9 M_\odot}\right)^{-5/3}\left(\frac{P_{\rm gw}}{1\ \rm yr}\right)^{8/3}$$

One can expects sources of sufficiently large strain and with almost constant frequency and amplitude over several years

Sources of low-frequency continuous GWs

- Examples of suspected binary SMBH
  (Valtonen et al, 2015; Graham et al, 2015; Yonemaru et al. 2016)

OJ287 $h_{\mathrm{OJ287}} \approx 2 \times 10^{-16}$ at 6 yr

PG 1302−102 $h_{\mathrm{PG1302\text{-}102}} \leq 5 \times 10^{-16}$ at 2.6 yr

M87, speculative $h_{\mathrm{M87}} < 1.3 \times 10^{-12} \left(\frac{P_{gw}^{\mathrm{M87}}}{1\,\mathrm{yr}} \right)^{-2/3}$

- strain is proportional to $\mathcal{M}^{5/3} / r$: larger masses help

- several SMBHs of up to $10^{10}\, M_{\odot}$ are reported

Gravitational Wave Spectrum

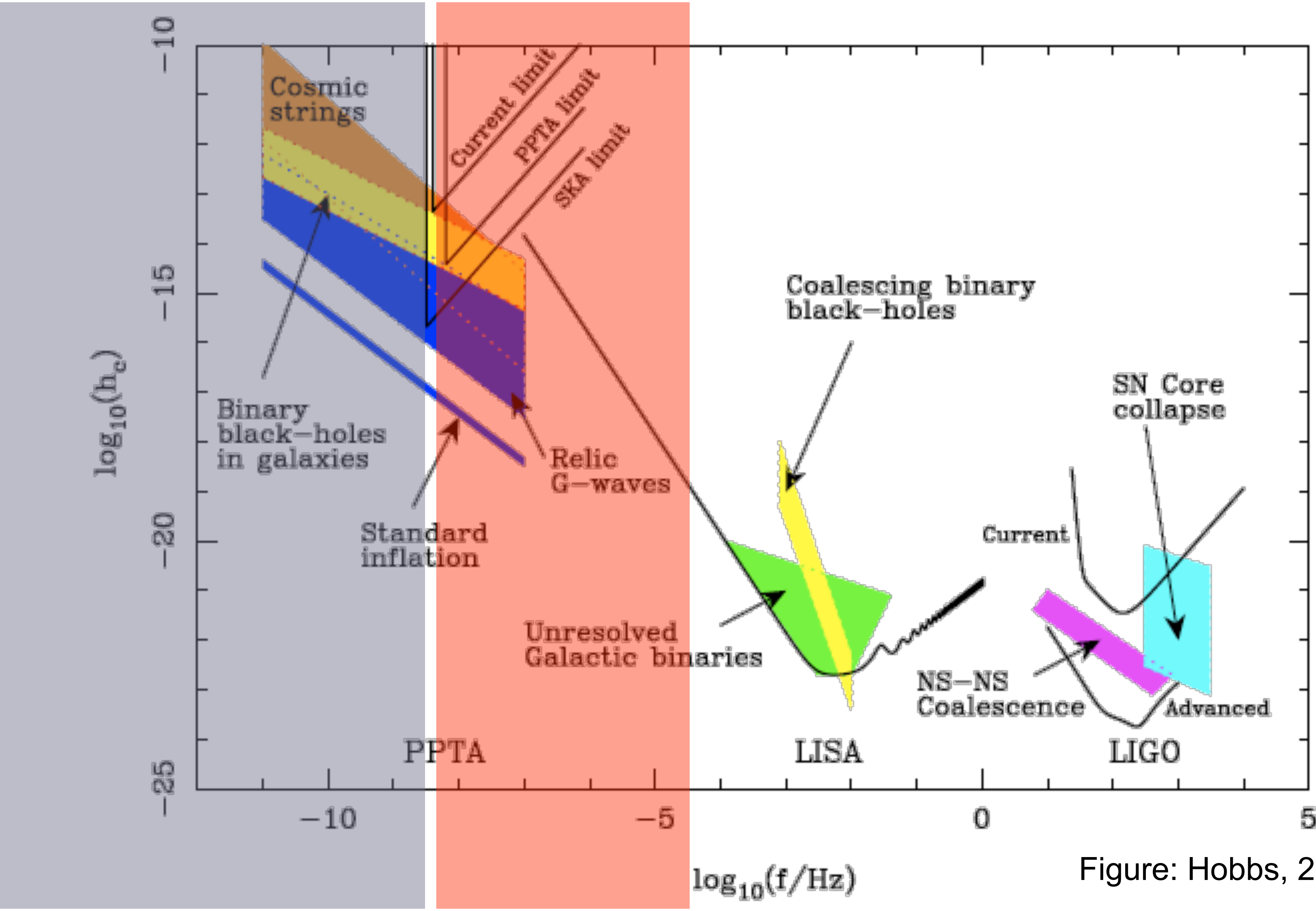


Figure: Hobbs, 2008

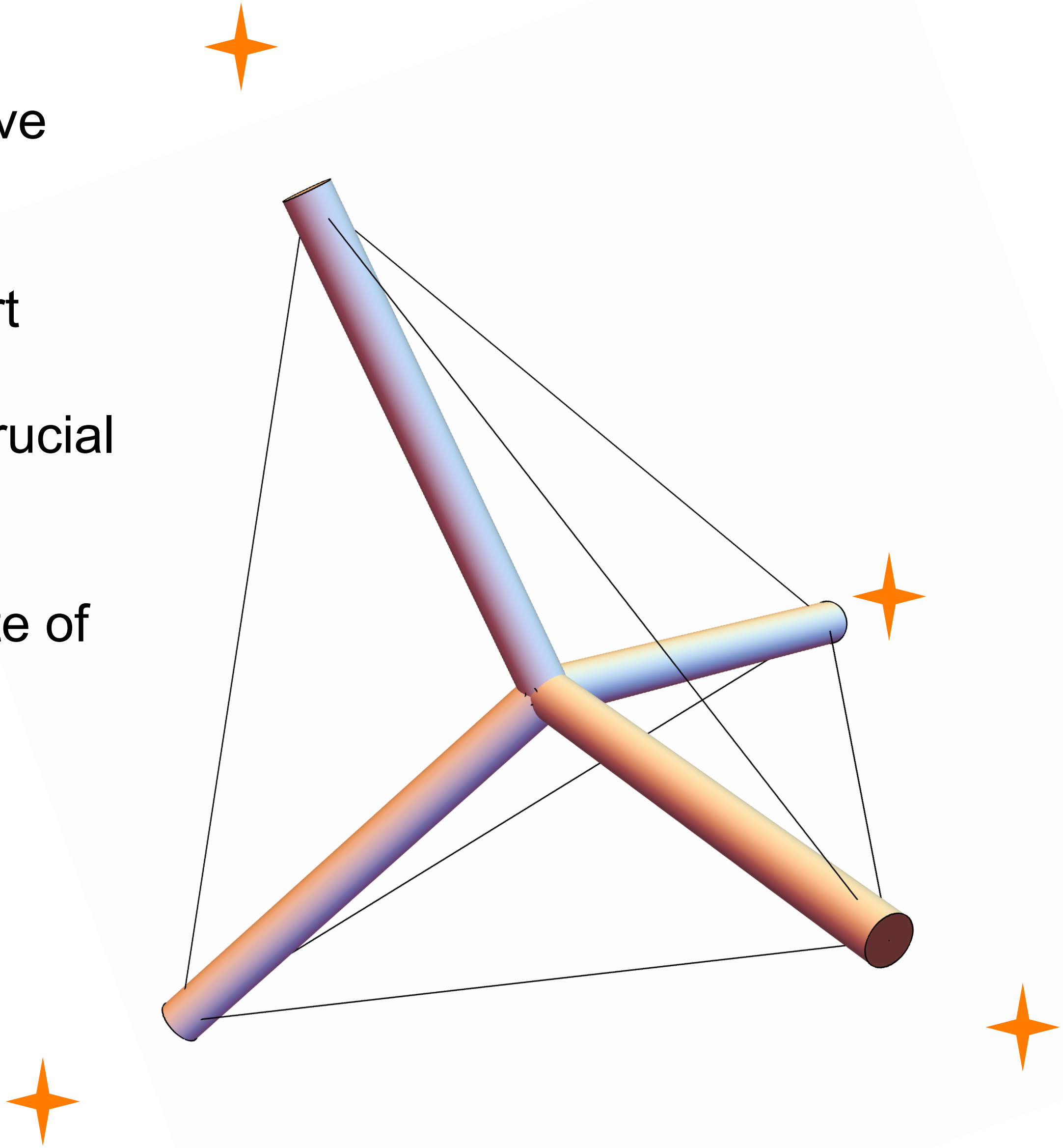
Tetra(hedron): astrometric gravitational observatory

Sufficient as a gravitational wave detector:

- 4 fixed stellar fields 120° apart

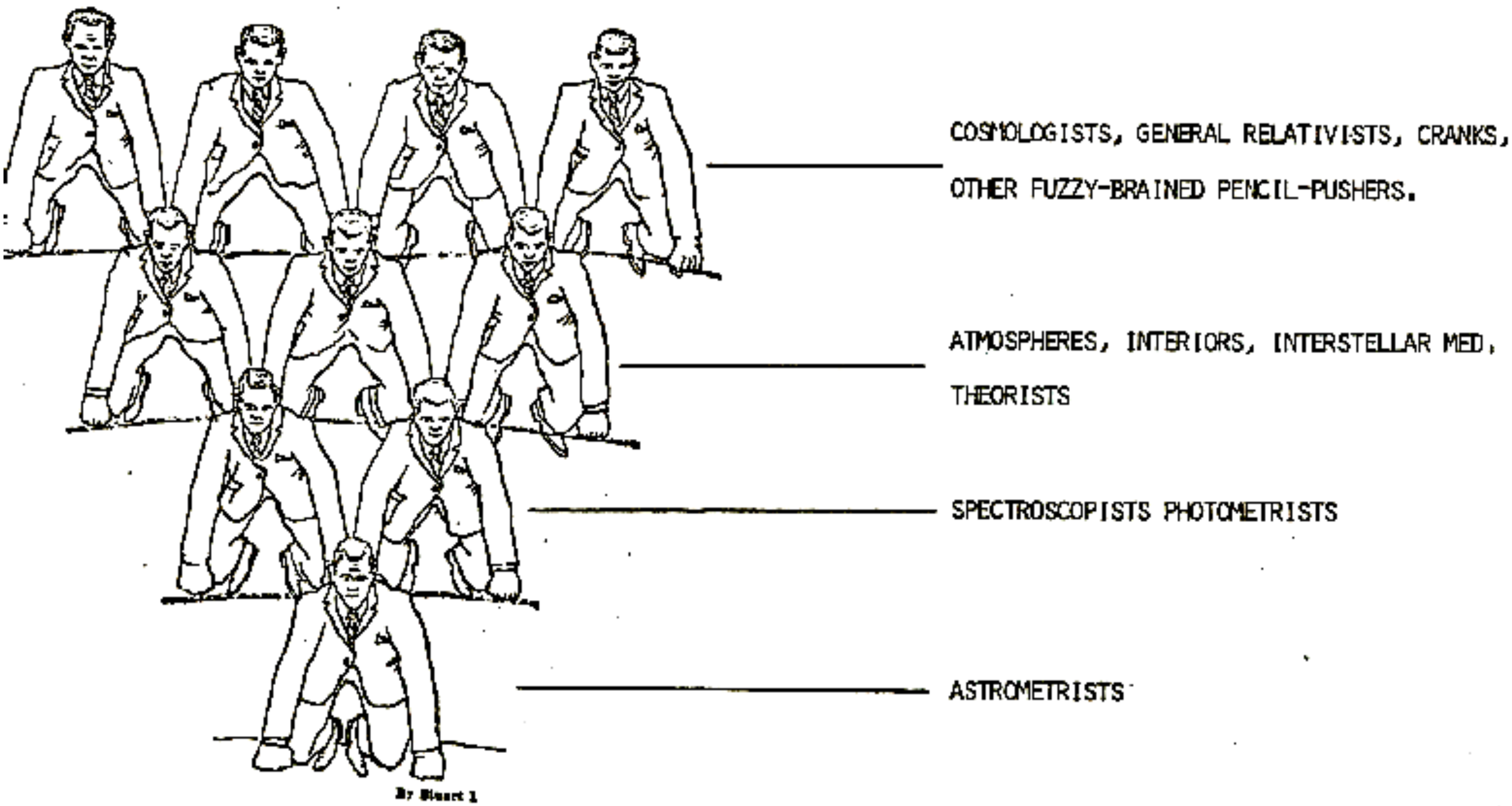
- stability of the instrument is crucial

It would also deliver an estimate of the PPN γ …



THE ASTRONOMICAL PYRAMID

ILLUSTRATING THE INTERDEPENDENCE OF THE VARIOUS AREAS OF STUDY



Ron Probst circa 1974

GET BACK TO BASICS -- SUPPORT ASTROMETRY

Get back to “basics” – support Astrometry